

Biot-Savart, ampère and magnetic sources

AP Physics C (Mechanics and E&M, calculus-based) · Lesson

Quick answer: The Biot-Savart law gives the field of a current element, $dB = (\mu_0/4\pi) I dl \times \hat{r}/r^2$, and integrating it yields $B = \mu_0 I / 2\pi d$ for a long wire and $\mu_0 I / 2R$ at a loop's centre. Ampère's law, $\oint B \cdot dl = \mu_0 I_{enc}$, gives symmetric fields quickly, including $B = \mu_0 n I$ inside a solenoid.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 12

What does the Biot-Savart law say?

Every small piece of current produces a small magnetic field around it, and the total field is the vector sum of all those pieces. The law gives the contribution of a single current element, and calculus adds them up for wires, loops and coils of any shape.

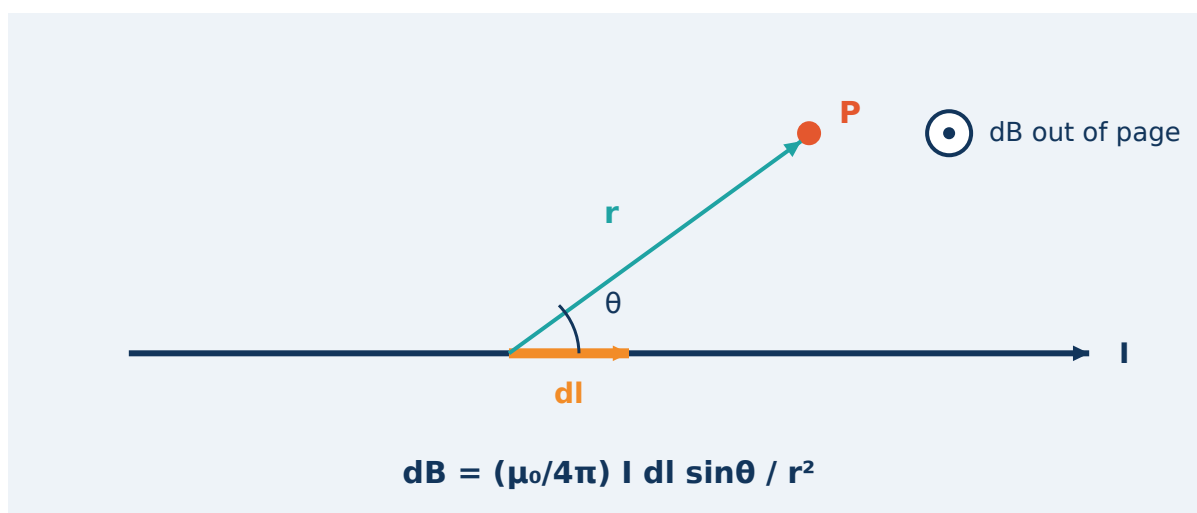
Key law

Biot-Savart law: $dB = (\mu_0/4\pi) I dl \times \hat{r} / r^2$, where dl is a length element along the current, r is the distance from the element to the field point and $\hat{r}$ is a unit vector from the element to that point.

Formula

Infinite straight wire: $B = \mu_0 I / (2\pi d)$ Circular loop, centre: $B = \mu_0 I / (2R)$ Loop, on axis: $B = \mu_0 R^2 / [2(x^2 + R^2)^{3/2}]$

B is the field (T), I is the current (A), d is the distance from the wire (m), R is the loop radius (m), x is the axial distance (m), and $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$.



The Biot-Savart geometry: the element dl and the vector r fix the direction of dB .

The cross product means dB is perpendicular to both dl and r . At a point on the line of the wire, dl is parallel to r , so $\sin \theta = 0$ and that element makes no field at all. For an infinite wire, the

integral of $\sin \theta$ gives $B = \mu_0 I / 2\pi d$, and the right-hand rule gives the direction: thumb along the current, fingers curl along B .

Worked example

Data: a loop of radius 5.0 cm carries 3.0 A. **Find** B at its centre.

Method: every element is a distance R from the centre and perpendicular to r , so $B = \mu_0 I / (2R) = (4\pi \times 10^{-7})(3.0)/(2 \times 0.050)$.

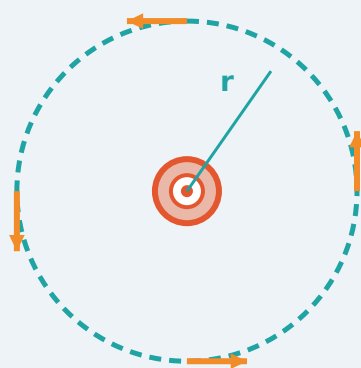
Answer: $B = 3.8 \times 10^{-5} \text{ T}$ (38 μT), directed along the loop's axis.

How does Ampère's law give magnetic fields?

When the current has a high degree of symmetry, you can skip the integration. Ampère's law links the circulation of B around any closed path to the current that passes through the path.

Definition

Ampère's law: the line integral of the magnetic field around a closed loop equals μ_0 times the net current through the loop: $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$. Currents are counted positive when they follow the right-hand rule for the direction of travel.



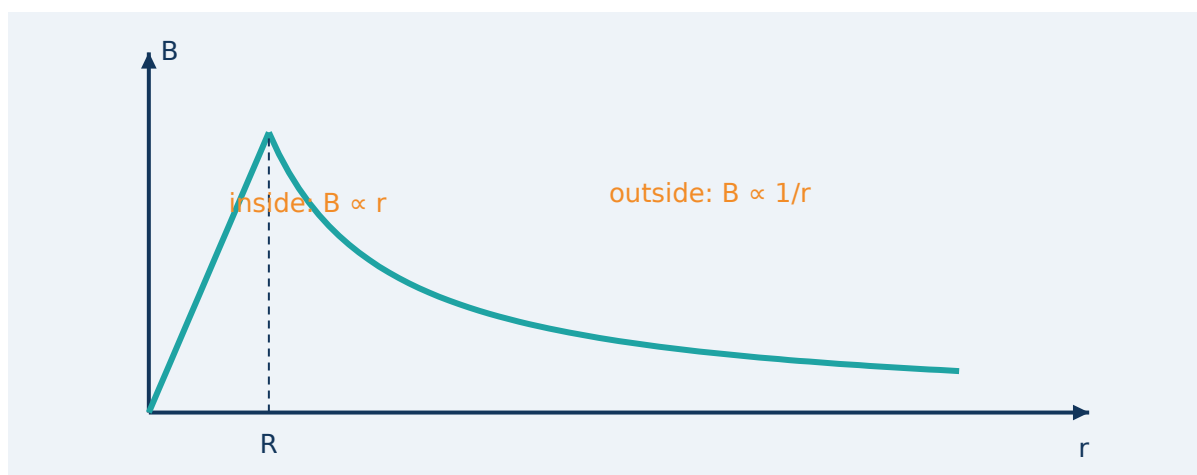
$$\oint \mathbf{B} \cdot d\mathbf{l} = B(2\pi r) = \mu_0 I$$

$$B = \mu_0 I / (2\pi r)$$

right-hand rule: thumb along I ,
fingers curl along B

A circular Amperian loop around a long wire: B is constant in magnitude and tangent to the loop.

Choose a loop on which B has constant magnitude and is either parallel or perpendicular to $d\mathbf{l}$. For a long cylindrical wire of radius R with uniform current density, the enclosed current inside the wire is $I(r^2/R^2)$, so $B = \mu_0 I r / (2\pi R^2)$ for $r < R$, and $B = \mu_0 I / (2\pi r)$ for $r > R$.



Inside a thick wire B grows linearly with r ; outside it falls as $1/r$.

Worked example

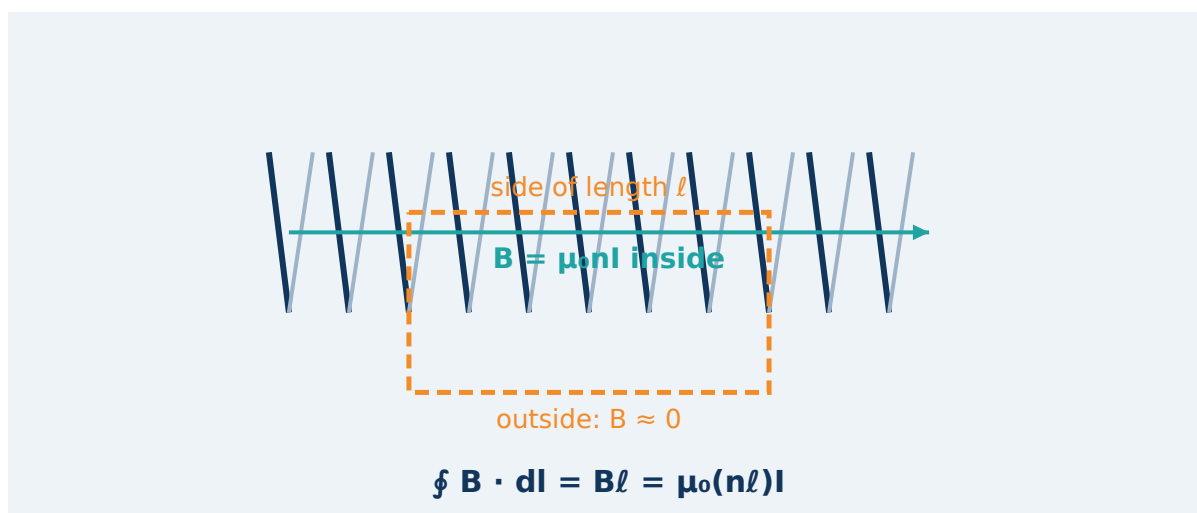
Data: a wire of radius 2.0 mm carries 5.0 A uniformly. **Find** B at $r = 1.0$ mm and at $r = 4.0$ mm.

Method: inside, $B = \mu_0 I r / (2\pi R^2) = (2 \times 10^{-7})(5.0)(1.0 \times 10^{-3}) / (2.0 \times 10^{-3})^2$. Outside, $B = \mu_0 I / (2\pi r)$.

Answer: $B = 0.25$ mT at 1.0 mm, and $B = 0.25$ mT at 4.0 mm, with a peak of 0.50 mT at the surface.

What is the field of a solenoid and the force between wires?

A long solenoid has n turns per metre. A rectangular loop with one side inside and one outside gives $B \ell = \mu_0(n\ell)I$, so the inside field is uniform and the outside field is almost zero. A toroid with N turns has $B = \mu_0 NI / (2\pi r)$ inside its core.

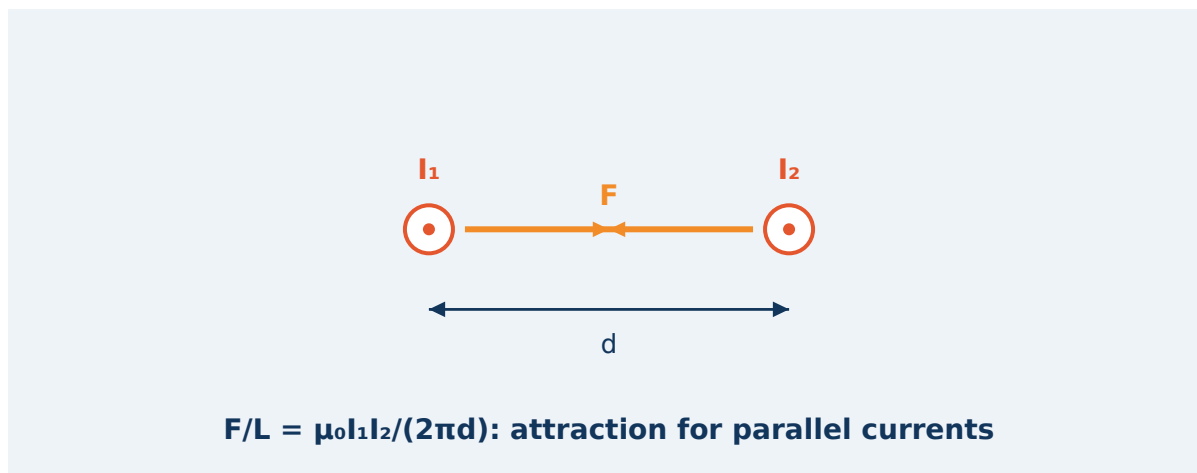


A rectangular Amperian loop on a solenoid gives $B = \mu_0 n I$ inside.

System	Magnetic field B
Long straight wire, distance d	$\mu_0 I / (2\pi d)$
Circular loop, centre	$\mu_0 I / (2R)$ (times N for N turns)

System	Magnetic field B
Long solenoid, inside	$\mu_0 n I$
Toroid, at radius r inside the core	$\mu_0 N I / (2\pi r)$
Thick wire, inside ($r < R$)	$\mu_0 I r / (2\pi R^2)$
Two wires, force per metre	$F/L = \mu_0 I_1 I_2 / (2\pi d)$

One long wire makes a field at the other; the other carries current in that field, so each feels a force $F = ILB$. Currents in the same direction attract and opposite currents repel.



Parallel currents attract: each wire feels the field of the other.

Worked example

Data: two parallel wires 0.10 m apart carry 20 A and 30 A in the same direction. **Find** the force per metre.

Method: $F/L = \mu_0 I_1 I_2 / (2\pi d) = (2 \times 10^{-7})(20)(30)/(0.10)$.

Answer: $F/L = 1.2 \times 10^{-3} \text{ N/m}$, attractive.

Common mistake

Ampère's law gives B only when symmetry lets you pull B out of the integral. It is always true, but not always useful. Do not use $B = \mu_0 n I$ near the ends of a short coil, where the field is not uniform.

Lina's tip

Before any calculation, draw the current and the loop, then apply the right-hand rule. A correct direction catches sign errors long before the arithmetic does.

Key points to remember

- $dB = (\mu_0/4\pi) I dl \times \hat{r} / r^2$; $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$.
- Long wire $B = \mu_0 I / (2\pi d)$; loop centre $B = \mu_0 N I / (2R)$; solenoid $B = \mu_0 n I$.
- Ampère's law: $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$, useful when symmetry fixes B.
- Thick wire: $B = \mu_0 I r / (2\pi R^2)$ inside, $\mu_0 I / (2\pi r)$ outside.

- Parallel wires: $F/L = \mu_0 I_1 I_2 / (2\pi d)$; same direction attracts.

Biot-Savart, Ampère and magnetic sources: frequently asked questions

When should I use Biot-Savart rather than Ampère's law?

Use Ampère's law when symmetry makes B constant on a simple loop, as for a long wire or solenoid. Use Biot-Savart for less symmetric shapes, such as a loop off-axis or a finite wire segment.

Why is the field zero along the line of a straight wire segment?

At such a point $d\mathbf{l}$ is parallel to $\mathbf{r}$, so the cross product $d\mathbf{l} \times \hat{\mathbf{r}}$ vanishes and every element contributes zero. The field is strongest perpendicular to the wire.

How do you get the direction of the magnetic field?

Point the thumb of your right hand along the current. Your curled fingers show how B circles the wire. For a loop or coil, curl the fingers along the current and the thumb gives B .

Why does B inside a thick wire grow with r ?

The Amperian loop of radius r encloses only the fraction r^2/R^2 of the current, so $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I (r^2/R^2)$. Dividing by $2\pi r$ gives $B = \mu_0 I r / (2\pi R^2)$.

Why do parallel currents attract?

Wire 1 creates a field at wire 2 whose direction, combined with the current in wire 2, produces a force $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$ toward wire 1. The same argument gives an equal force back on wire 1.