

Biot-Savart, ampère and magnetic sources

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the law used, the substitution and the result with its unit in bold. Compare your choice of Amperian loop and your direction reasoning with the model, because a wrong enclosed current or a flipped direction is the commonest source of lost marks.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 12

Problem 1 Direction of B

1. The field lines are **concentric circles** around the wire.
2. With the thumb out of the page the fingers curl anticlockwise, so B at a point on the right points **up the page**.

Problem 2 Long wire field

1. $B = \mu_0 I / (2\pi d) = (2 \times 10^{-7})(10)/0.050 = \mathbf{4.0 \times 10^{-5} \text{ T}}$.
2. $B \propto 1/d$, so at 10 cm it is **half**, 20 μT .

Problem 3 Solenoid field

1. $n = 500/0.25 = \mathbf{2000 \text{ turns/m}}$.
2. $B = \mu_0 n I = (4\pi \times 10^{-7})(2000)(2.0) = \mathbf{5.0 \times 10^{-3} \text{ T}}$.

Problem 4 Loop at its centre

1. $B = \mu_0 N I / (2R) = (4\pi \times 10^{-7})(20)(0.50)/(2 \times 0.040) = \mathbf{1.6 \times 10^{-4} \text{ T}}$.

Problem 5 Force between wires

1. $F/L = \mu_0 I_1 I_2 / (2\pi d) = (2 \times 10^{-7})(25)(25)/0.020 = \mathbf{2.5 \times 10^{-4} \text{ N/m}}$.
2. Opposite currents **repel**.

Problem 6 Reading a B-r graph

1. $I = 2\pi r B / \mu_0 = (0.020)(1.0 \times 10^{-4}) / (2 \times 10^{-7}) = \mathbf{10 \text{ A}}$.
2. $B \propto 1/r$, so at 10 cm $B = 100 \times 2.0/10 = \mathbf{20 \mu\text{T}}$.

Problem 7 Ampère's law statement

1. The line integral of B around a closed loop equals μ_0 times the net current through it.
2. $I_{\text{net}} = 3.0 - 1.0 = 2.0 \text{ A}$, so $\oint \mathbf{B} \cdot d\mathbf{l} = (4\pi \times 10^{-7})(2.0) = \mathbf{2.5 \times 10^{-6} \text{ T} \cdot \text{m}}$.

Problem 8 Inside a thick wire

- $B = \mu_0 I r / (2\pi R^2) = (2 \times 10^{-7})(5.0)(1.0 \times 10^{-3}) / (4.0 \times 10^{-6}) = \mathbf{2.5 \times 10^{-4} \text{ T}}$.
- At the surface $B = (2 \times 10^{-7})(5.0) / (2.0 \times 10^{-3}) = \mathbf{5.0 \times 10^{-4} \text{ T}}$.
- $B = (2 \times 10^{-7})(5.0) / (4.0 \times 10^{-3}) = \mathbf{2.5 \times 10^{-4} \text{ T}}$.

Problem 9 Finite wire segment

- $\sin \theta = 0.30 / \sqrt{(0.30^2 + 0.10^2)} = 0.30 / 0.316 = 0.949$.
- $B = (10^{-7})(4.0)(2 \times 0.949) / 0.10 = \mathbf{7.6 \times 10^{-6} \text{ T}}$.
- An infinite wire gives $(2 \times 10^{-7})(4.0) / 0.10 = 8.0 \text{ } \mu\text{T}$, so the finite wire gives **95 %** of it.

Problem 10 Loop on its axis

- $B = \mu_0 I / (2R) = (4\pi \times 10^{-7})(3.0) / 0.10 = \mathbf{3.8 \times 10^{-5} \text{ T}}$.
- $B = \mu_0 I R^2 / [2(x^2 + R^2)^{3/2}]$: $x^2 + R^2 = 0.0169$, its $3/2$ power is 2.197×10^{-3} , so $B = (3.77 \times 10^{-6})(0.0025) / (2 \times 2.197 \times 10^{-3}) = \mathbf{2.1 \times 10^{-6} \text{ T}}$.
- Ratio = $2.145 / 37.7 = \mathbf{0.057}$.

Problem 11 Solenoid by Ampère

- $I_{\text{enc}} = nI = (3000)(0.10)(0.80) = \mathbf{240 \text{ A}}$ (the sum of the turns' currents).
- $B\ell = \mu_0 nI\ell$ gives $B = (4\pi \times 10^{-7})(3000)(0.80) = \mathbf{3.0 \times 10^{-3} \text{ T}}$.
- Along bc and da the field is perpendicular to the path (or zero), so $B \cdot d\mathbf{l} = 0$.

Problem 12 Toroid

- $B = \mu_0 NI / (2\pi r) = (2 \times 10^{-7})(400)(1.2) / 0.10 = \mathbf{9.6 \times 10^{-4} \text{ T}}$.
- $B(0.080) = \mathbf{1.2 \times 10^{-3} \text{ T}}$ and $B(0.12) = \mathbf{8.0 \times 10^{-4} \text{ T}}$.

Problem 13 Opposite currents

- At M the left wire's field (current out) points down the page and the right wire's field (current in) also points down, so they **point the same way**.
- Each $B = (2 \times 10^{-7})(6.0) / 0.10 = 1.2 \times 10^{-5} \text{ T}$, so the total is $\mathbf{2.4 \times 10^{-5} \text{ T}}$.
- With the same direction the fields cancel and $\mathbf{B = 0}$.

Problem 14 Force over a length

- $F = (F/L)L = (1.2 \times 10^{-3})(2.5) = \mathbf{3.0 \times 10^{-3} \text{ N}}$.
- The wires **attract**, so each force points toward the other wire.

Problem 15 A moving proton

- $B = (10^{-7})(1.6 \times 10^{-19})(2.0 \times 10^6) / (1.0 \times 10^{-2})^2 = \mathbf{3.2 \times 10^{-16} \text{ T}}$.

2. A 1 A current is about 6×10^{18} elementary charges passing every point each second, and such a wire gives 2×10^{-5} T at 1 cm, so one proton contributes very little.

Problem 16 Semicircular wire

1. On each straight section $d\mathbf{l}$ is parallel to $\mathbf{r}$, so $d\mathbf{l} \times \hat{\mathbf{r}} = 0$ at O.
2. On the arc every $d\mathbf{l}$ is perpendicular to $\mathbf{r}$, so $B = (\mu_0 I / 4\pi)(\pi R) / R^2 = \mu_0 I / (4R) = (4\pi \times 10^{-7})(6.0) / (4 \times 0.080) = \mathbf{2.4 \times 10^{-5} \text{ T}}$.

Problem 17 Coaxial cable

1. $B = \mu_0 I / (2\pi r) = (2 \times 10^{-7})(2.0) / (3.0 \times 10^{-3}) = \mathbf{1.3 \times 10^{-4} \text{ T}}$.
2. The net enclosed current is $2.0 - 2.0 = 0$, so $\mathbf{B = 0}$ at 8.0 mm.
3. Outside the cable the opposite currents cancel, so there is no external field to disturb nearby signals.

Problem 18 Non-uniform current

1. $I = \int J 2\pi r dr = (2\pi J_0 / R) \int_0^R r^2 dr = 2\pi J_0 R^2 / 3 = 2\pi(4.0 \times 10^5)(9.0 \times 10^{-6}) / 3 = \mathbf{7.5 \text{ A}}$.
2. $I_{\text{enc}} = 2\pi J_0 r^3 / (3R)$, so $B = \mu_0 I_{\text{enc}} / (2\pi r) = \mathbf{\mu_0 J_0 r^2 / (3R)}$.
3. $B(R) = \mu_0 J_0 R / 3 = (1.2566 \times 10^{-6})(4.0 \times 10^5)(3.0 \times 10^{-3}) / 3 = \mathbf{5.0 \times 10^{-4} \text{ T}}$, and $B(R/2) = B(R) / 4 = \mathbf{1.3 \times 10^{-4} \text{ T}}$.

Problem 19 Loop near a wire

1. $F_1 = \mu_0 I_1 I_2 L / (2\pi d_1) = (2 \times 10^{-7})(20)(5.0)(0.10) / 0.050 = \mathbf{4.0 \times 10^{-5} \text{ N}}$ toward the wire.
2. The far side is at 0.25 m and its current is opposite: $F_2 = (2 \times 10^{-7})(20)(5.0)(0.10) / 0.25 = \mathbf{8.0 \times 10^{-6} \text{ N}}$, away.
3. The net force is $4.0 \times 10^{-5} - 0.8 \times 10^{-5} = \mathbf{3.2 \times 10^{-5} \text{ N toward the wire}}$. The side forces cancel.

Problem 20 Spot the error

1. $B = \mu_0 I / (2\pi r)$ holds only outside, where all the current is enclosed. Inside, only the fraction r^2 / R^2 is enclosed.
2. Emma: $(2 \times 10^{-7})(8.0) / (2.0 \times 10^{-3}) = \mathbf{8.0 \times 10^{-4} \text{ T}}$.
3. Correct: $B = \mu_0 I r / (2\pi R^2) = (2 \times 10^{-7})(8.0)(2.0 \times 10^{-3}) / (1.6 \times 10^{-5}) = \mathbf{2.0 \times 10^{-4} \text{ T}}$, a quarter of her value.