

Rotational energy, rolling and angular momentum

AP Physics 1 (algebra-based) · Lesson

Quick answer: Angular momentum is $L = I\omega$, and it stays constant when no net external torque acts, so a spinner who pulls mass inward spins faster. Rotating objects also carry kinetic energy $K = \frac{1}{2}I\omega^2$, and a rolling object has both $\frac{1}{2}mv^2$ and $\frac{1}{2}I\omega^2$ with $v = R\omega$.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 6

What is rotational kinetic energy?

A spinning flywheel stores energy even though its centre never moves. Every piece of the wheel is travelling in a circle, so each piece has kinetic energy. Adding them all up gives a compact result that mirrors the straight-line formula.

Definition

Rotational kinetic energy is the energy of an object due to its spin: $K_{\text{rot}} = \frac{1}{2}I\omega^2$, where I is the rotational inertia about the axis (kg m^2) and ω is the angular speed (rad/s). A net torque doing work $W = \tau\Delta\theta$ changes it: $W_{\text{net}} = \Delta K_{\text{rot}}$.

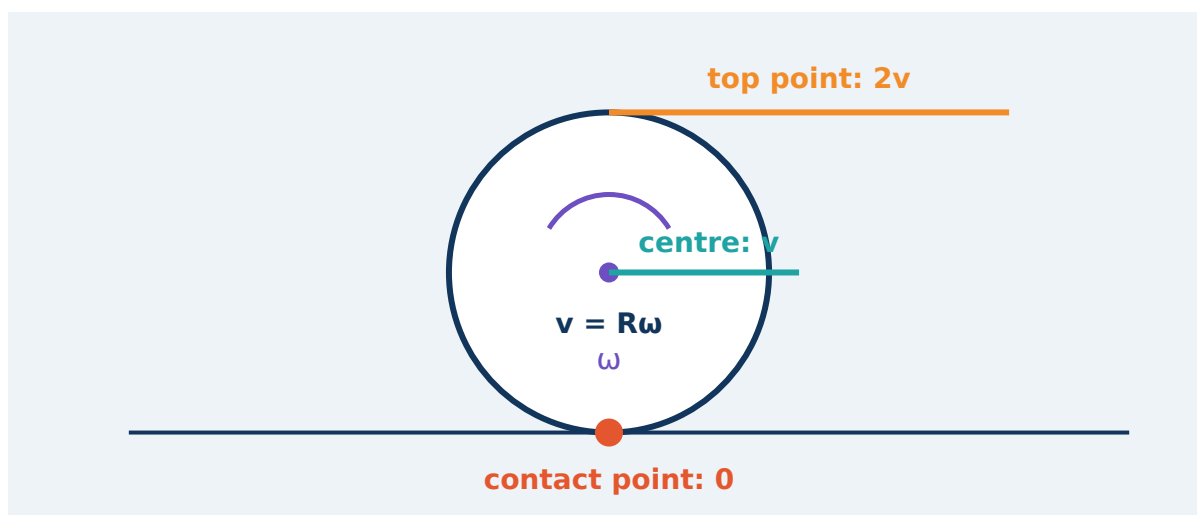
Worked example 1

A flywheel has $I = 0.50 \text{ kg m}^2$ and turns at 40 rad/s . Find its rotational kinetic energy, and the work needed to bring it from rest to that speed.

Method: $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.50 \times (40)^2 = 4.0 \times 10^2 \text{ J}$. Starting from rest, the work done by the torque equals the gain in K : $W = 400 \text{ J}$.

How do rolling objects store energy?

When a wheel rolls without slipping, its centre moves with speed v while the wheel spins with angular speed ω . The two are tied together by $v = R\omega$, because the contact point is momentarily at rest, as Figure 1 shows.



Rolling without slipping links the straight-line speed to the spin: $v = R\omega$.

A rolling object therefore has two kinetic energies, translation of the centre of mass and rotation about it: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. Because part of the energy goes into spinning, a rolling object reaches a lower speed than a sliding one released from the same height.

Key law

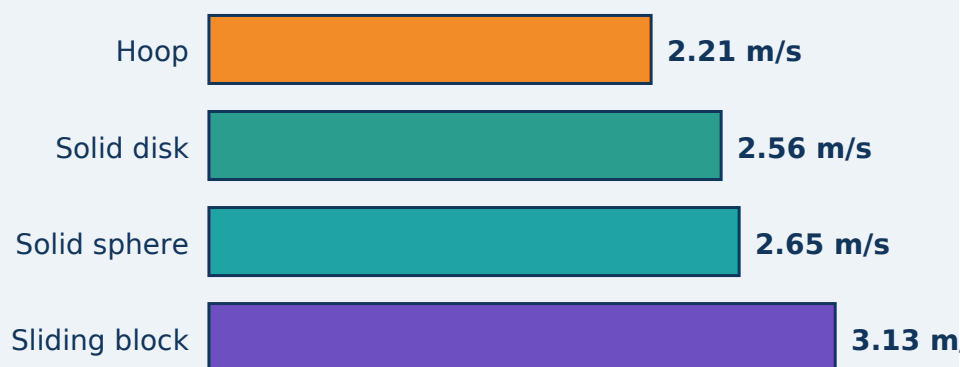
Rolling without slipping: $v = R\omega$ and $K_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2(1 + I/mR^2)$. From rest down a height h , energy conservation gives $mgh = \frac{1}{2}mv^2(1 + I/mR^2)$.

Worked example 2

A solid disk ($I = \frac{1}{2}mR^2$) rolls without slipping from rest down a ramp of vertical height 0.80 m. Find its speed at the bottom.

Method: $I/mR^2 = \frac{1}{2}$, so $mgh = \frac{1}{2}mv^2 \times 1.5 = \frac{3}{4}mv^2$. Then $v = \sqrt{(4gh/3)} = \sqrt{(4 \times 9.81 \times 0.80 \div 3)} = \sqrt{10.46} = \mathbf{3.2 \text{ m/s}}$. A frictionless sliding block would reach $\sqrt{(2gh)} = 4.0 \text{ m/s}$.

Speed at the bottom of a ramp 0.50 m high



Rolling objects share their energy between moving and spinning, so each is slower than the frictionless sliding block. The more mass near the axis, the faster the roll.

What is angular momentum, and when is it conserved?

Linear momentum measures how hard it is to stop a moving object. Angular momentum measures how hard it is to stop a spinning one.

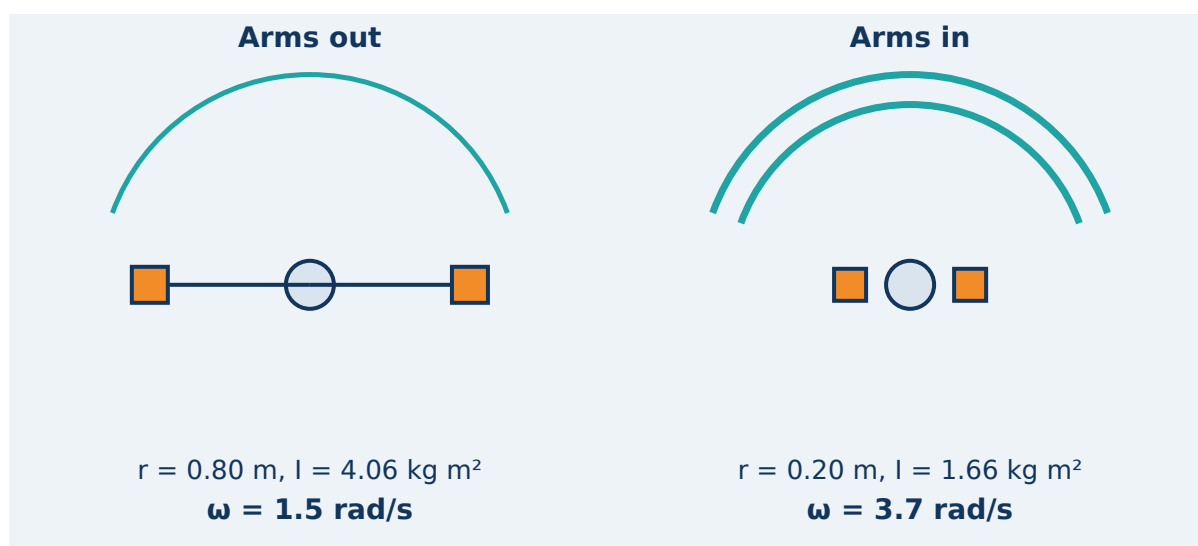
Definition

For a rigid object rotating about a fixed axis, angular momentum is $L = I\omega$, measured in $\text{kg m}^2/\text{s}$. For a single point mass moving with speed v at a perpendicular distance r from a chosen point, $L = mvr$.

Key law

Conservation of angular momentum: if the net external torque on a system is zero, its total angular momentum stays constant, so $I_i\omega_i = I_f\omega_f$. A net torque changes L at the rate $\tau_{\text{net}} = \Delta L/\Delta t$.

A skater or a person on a rotating stool can change I by moving their mass in or out. With no external torque, L is fixed, so a smaller I must be matched by a larger ω .



Pulling the masses inwards lowers I , so ω rises while $L = I\omega$ stays constant. The drawing is not to scale.

Worked example 3

In Figure 3 a person ($I = 1.5 \text{ kg m}^2$) on a frictionless stool holds two 2.0 kg masses 0.80 m from the axis, turning at 1.5 rad/s . They pull the masses in to 0.20 m . Find the new angular speed and the change in kinetic energy.

Method: $I_i = 1.5 + 2 \times 2.0 \times (0.80)^2 = 4.06 \text{ kg m}^2$; $I_f = 1.5 + 2 \times 2.0 \times (0.20)^2 = 1.66 \text{ kg m}^2$. $\omega_f = I_i\omega_i/I_f = 4.06 \times 1.5 \div 1.66 = \mathbf{3.7 \text{ rad/s}}$. $K_i = \frac{1}{2} \times 4.06 \times (1.5)^2 = 4.6 \text{ J}$ and $K_f = \frac{1}{2} \times 1.66 \times (3.67)^2 = 11.2 \text{ J}$, so K rises by about **6.6 J**, supplied by the person's muscles.

Quantity	Straight-line motion	Rotation about a fixed axis
Inertia	mass m	rotational inertia I
Cause of change	net force F	net torque τ

Quantity	Straight-line motion	Rotation about a fixed axis
Newton's second law	$F = ma$	$\tau = I\alpha$
Momentum	$p = mv$	$L = I\omega$
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$

Common mistake

Assuming that conserved angular momentum means conserved kinetic energy. When the masses in Figure 3 are pulled in, L stays constant but K increases because the person does work. In a sticky collision, the reverse happens: L is conserved but K decreases.

Lina's tip

Make a before-and-after table with I and ω for each state. Use $L = I\omega$ first to find ω_f , then compute K for both states. If K changed, ask where the energy came from or went.

Key points to remember

- Rotational kinetic energy: $K_{\text{rot}} = \frac{1}{2}I\omega^2$, in joules
- Angular momentum: $L = I\omega$ (rigid body) or $L = mvr$ (point mass), in $\text{kg m}^2/\text{s}$
- Rolling without slipping: $v = R\omega$ and $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- If net external torque is zero, L is conserved: $I_i\omega_i = I_f\omega_f$
- Work by a torque: $W = \tau\Delta\theta$, and power $P = \tau\omega$

Rotational energy, rolling and angular momentum: frequently asked questions

Why does a skater spin faster when she pulls in her arms?

There is almost no external torque on her, so angular momentum $L = I\omega$ stays constant. Pulling her arms in reduces her rotational inertia I , so ω must increase to keep L the same. Her kinetic energy rises because her muscles do work pulling the arms inward.

What does rolling without slipping mean?

It means the point of the wheel touching the ground is momentarily at rest relative to the ground. The centre of the wheel then moves at $v = R\omega$. Static friction makes this possible but does no work and loses no energy.

Why does a solid disk beat a hoop down a ramp?

Both start with the same potential energy, but the hoop has more of its mass at the rim, so more rotational inertia and more of its energy must go into spinning. The disk puts more energy into translation, so its centre moves faster.

Is angular momentum always conserved?

Only for a system with zero net external torque about the chosen axis. Gravity acting on a stool or an outside push, for example, can change L . Internal forces and torques never change

a system's total angular momentum.

What is the difference between $L = I\omega$ and $L = mvr$?

$L = I\omega$ applies to a rigid object turning about a fixed axis. $L = mvr$ gives the angular momentum of a point mass moving past a chosen point, with r the perpendicular distance from that point to the line of motion. Both describe the same physical quantity.