

Rotational energy, rolling and angular momentum

AP Physics 1 (algebra-based) · Worksheet answers

Each answer shows the law used, the substitution and the final result in bold. For conservation problems, check L before and after, and then compare kinetic energies to see whether energy was added or lost.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 6

Problem 1 Spinning flywheel energy

- $K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.50 \times (40)^2 = \mathbf{4.0 \times 10^2 \text{ J}}$.
- At 80 rad/s: $K = \frac{1}{2} \times 0.50 \times (80)^2 = \mathbf{1.6 \times 10^3 \text{ J}}$, four times larger because $K \propto \omega^2$.

Problem 2 Revolutions per minute

- $\omega = 120 \times 2\pi \div 60 = \mathbf{12.6 \text{ rad/s}}$.
- $K = \frac{1}{2} \times 0.20 \times (12.57)^2 = \mathbf{16 \text{ J}}$ (15.8 J).

Problem 3 Angular momentum of a turntable

- $L = I\omega = 0.40 \times 15 = \mathbf{6.0 \text{ kg m}^2/\text{s}}$.
- $L = 0.40 \times 10 = \mathbf{4.0 \text{ kg m}^2/\text{s}}$.

Problem 4 Rolling wheel

- $\omega = v/R = 7.0 \div 0.35 = \mathbf{20 \text{ rad/s}}$.
- $20 \div 2\pi = \mathbf{3.2 \text{ revolutions per second}}$.

Problem 5 A skater pulls in her arms

- $I_i\omega_i = I_f\omega_f$; $\omega_f = 3.0 \times 2.0 \div 1.5 = \mathbf{4.0 \text{ rad/s}}$.
- Angular momentum** is conserved, because there is no net external torque on the skater.

Problem 6 Work done by a torque

- $W = \tau\Delta\theta = 4.0 \times 5.0 = \mathbf{20 \text{ J}}$.
- $W = \frac{1}{2}I\omega^2$; $\omega = \sqrt{(2W/I)} = \sqrt{(2 \times 20 \div 0.40)} = \sqrt{100} = \mathbf{10 \text{ rad/s}}$.

Problem 7 Skater's energy

1. $\omega_f = 2.4 \times 3.0 \div 0.80 = \mathbf{9.0 \text{ rad/s}}$.
2. $K_i = \frac{1}{2} \times 2.4 \times 9.0 = \mathbf{10.8 \text{ J}}$; $K_f = \frac{1}{2} \times 0.80 \times 81 = \mathbf{32.4 \text{ J}}$.
3. The skater's muscles do 21.6 J of work pulling her arms inwards, against the outward (centripetal) requirement of the circular motion.

Problem 8 Rolling ball

1. $K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.50 \times 9.0 = \mathbf{2.25 \text{ J}}$.
2. $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times (2/5)mR^2 \times (v/R)^2 = (1/5)mv^2 = 0.20 \times 0.50 \times 9.0 = \mathbf{0.90 \text{ J}}$.
3. Fraction = $0.90 \div 3.15 = \mathbf{29 \% (2/7)}$.

Problem 9 Rolling down a ramp

1. Disk: $v = \sqrt{4gh/3} = \sqrt{4 \times 9.81 \times 1.5 \div 3} = \sqrt{19.62} = \mathbf{4.4 \text{ m/s}}$.
2. Hoop ($I/mR^2 = 1$): $mgh = mv^2$, $v = \sqrt{gh} = \sqrt{14.7} = \mathbf{3.8 \text{ m/s}}$.
3. The **disk** arrives first because it reaches a higher speed.

Problem 10 Motor and a disk

1. $\alpha = \tau/I = 2.5 \div 0.25 = 10 \text{ rad/s}^2$, so $\omega = \alpha t = \mathbf{80 \text{ rad/s}}$.
2. $K = \frac{1}{2} \times 0.25 \times (80)^2 = \mathbf{800 \text{ J}}$; $P_{\text{avg}} = 800 \div 8.0 = \mathbf{100 \text{ W}}$.
3. $P = \tau\omega = 2.5 \times 80 = \mathbf{200 \text{ W}}$, twice the average because power grows linearly with ω .

Problem 11 Clay on a turntable

1. $I_{\text{disk}} = \frac{1}{2} \times 2.0 \times (0.30)^2 = \mathbf{0.090 \text{ kg m}^2}$; $I_{\text{clay}} = 0.20 \times (0.25)^2 = \mathbf{0.0125 \text{ kg m}^2}$.
2. $\omega_f = 0.090 \times 5.0 \div 0.1025 = \mathbf{4.4 \text{ rad/s}}$.
3. $K_i = \frac{1}{2} \times 0.090 \times 25 = 1.125 \text{ J}$; $K_f = \frac{1}{2} \times 0.1025 \times (4.39)^2 = 0.988 \text{ J}$. Lost = $\mathbf{0.14 \text{ J}}$ (about 12 %).

Problem 12 Angular momentum of a moving ball

1. $L = mvr = 0.20 \times 5.0 \times 0.40 = \mathbf{0.40 \text{ kg m}^2/\text{s}}$ (r is the perpendicular distance from O to the line of motion).
2. No net torque acts about O (no force, ignoring gravity and friction), and the perpendicular distance stays 0.40 m, so L is constant.

Problem 13 Storing energy in a flywheel

1. $\omega = 3000 \times 2\pi \div 60 = \mathbf{314 \text{ rad/s}}$.
2. $K = \frac{1}{2} \times 2.0 \times (314.2)^2 = \mathbf{9.9 \times 10^4 \text{ J}}$ (98.7 kJ).

3. $t = E/P = 98\,700 \div 100 = \mathbf{9.9 \times 10^2 \text{ s}}$, about 16 minutes.

Problem 14 Ranking rolling objects

- $v = \sqrt{(2gh/(1 + I/mR^2))}$: hoop $\sqrt{(gh)}$; disk $\sqrt{(4gh/3)}$; sphere $\sqrt{(10gh/7)}$.
- $gh = 4.905 \text{ m}^2/\text{s}^2$: hoop = **2.2 m/s**; disk = $\sqrt{6.54} = \mathbf{2.6 \text{ m/s}}$; sphere = $\sqrt{7.007} = \mathbf{2.6 \text{ m/s}}$ (2.65 m/s).
- Sphere first, then disk, then hoop**, since a smaller I/mR^2 means less energy in spin.

Problem 15 Putty hits a rod

- $L = mvr = 0.20 \times 3.0 \times 0.60 = \mathbf{0.36 \text{ kg m}^2/\text{s}}$.
- $I = (1/3)ML^2 + mr^2 = (1/3) \times 1.0 \times 0.36 + 0.20 \times 0.36 = 0.12 + 0.072 = \mathbf{0.19 \text{ kg m}^2}$.
- $\omega = L/I = 0.36 \div 0.192 = \mathbf{1.9 \text{ rad/s}}$. $K_i = \frac{1}{2} \times 0.20 \times 9.0 = 0.90 \text{ J}$; $K_f = \frac{1}{2} \times 0.192 \times (1.875)^2 = 0.34 \text{ J}$; lost = **0.56 J**.

Problem 16 Acceleration on a slope

- $mg \sin\theta - f = ma$.
- Torque of friction about the centre: $fR = I\alpha = \frac{1}{2}mR^2 (a/R)$, so $f = \frac{1}{2}ma$.
- $mg \sin\theta = (3/2)ma$, so $a = (2/3)g \sin\theta = (2/3) \times 9.81 \times 0.50 = \mathbf{3.3 \text{ m/s}^2}$ (3.27). $f = \frac{1}{2} \times 2.0 \times 3.27 = \mathbf{3.3 \text{ N}}$, directed up the slope.

Problem 17 Rolling hoop up a hill

- $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mR^2(v/R)^2 = mv^2 = 1.5 \times 16 = \mathbf{24 \text{ J}}$.
- $mgh = 24 \text{ J}$, $h = 24 \div (1.5 \times 9.81) = \mathbf{1.6 \text{ m}}$.
- Sliding block: $h = v^2/(2g) = 16 \div 19.62 = \mathbf{0.82 \text{ m}}$, only half as high, because the rolling hoop must keep half of its energy in rotation.

Problem 18 Jumping onto a roundabout

- $L_i = I\omega + mvR = 200 \times 1.2 + 30 \times 4.0 \times 2.0 = 240 + 240 = \mathbf{480 \text{ kg m}^2/\text{s}}$.
- $I_f = 200 + 30 \times (2.0)^2 = 320 \text{ kg m}^2$; $\omega_f = 480 \div 320 = \mathbf{1.5 \text{ rad/s}}$.
- $K_i = \frac{1}{2} \times 200 \times (1.2)^2 + \frac{1}{2} \times 30 \times (4.0)^2 = 144 + 240 = 384 \text{ J}$; $K_f = \frac{1}{2} \times 320 \times (1.5)^2 = 360 \text{ J}$; lost = **24 J**.

Problem 19 Spot the error

- $\omega_f = I_i\omega_i/I_f = 3.0 \times 2.0 \div 1.0 = \mathbf{6.0 \text{ rad/s}}$.
- $K_i = \frac{1}{2} \times 3.0 \times 4.0 = \mathbf{6.0 \text{ J}}$; $K_f = \frac{1}{2} \times 1.0 \times 36 = \mathbf{18 \text{ J}}$.
- Constant L does not imply constant K . $K = L^2/2I$, so reducing I raises K ; here the skater's muscles supply the extra 12 J.

Problem 20 Testing conservation of angular momentum

1. Gradient = $(11.9 - 3.1) \div (8.0 - 2.0) = 8.8 \div 6.0 = \mathbf{1.47}$.
2. $\omega = L \times (1/I)$, so the gradient is the angular momentum: **1.47 kg m²/s**.
3. Difference = $(1.50 - 1.47) \div 1.50 = \mathbf{2\%}$, which supports conservation of L within the scatter of the data.