

Angular momentum and rolling motion

AP Physics C (Mechanics and E&M, calculus-based) · Lesson

Quick answer: Angular momentum is the rotational equivalent of momentum: $L = I\omega$ for a rigid body, or $L = r \times p$ for a particle. It is conserved when the net external torque is zero. For rolling without slipping, $v = R\omega$, and the kinetic energy splits into translational and rotational parts.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 6

What is angular momentum?

Linear momentum measures how hard it is to stop a moving object. Angular momentum plays the same role for spinning. For a single particle of mass m moving with velocity v at position r from a chosen point, $L = r \times p$, so $L = m v r \sin\theta$. For a rigid body turning about a fixed axis, $L = I\omega$, where $I = \int r^2 dm$.

Definition

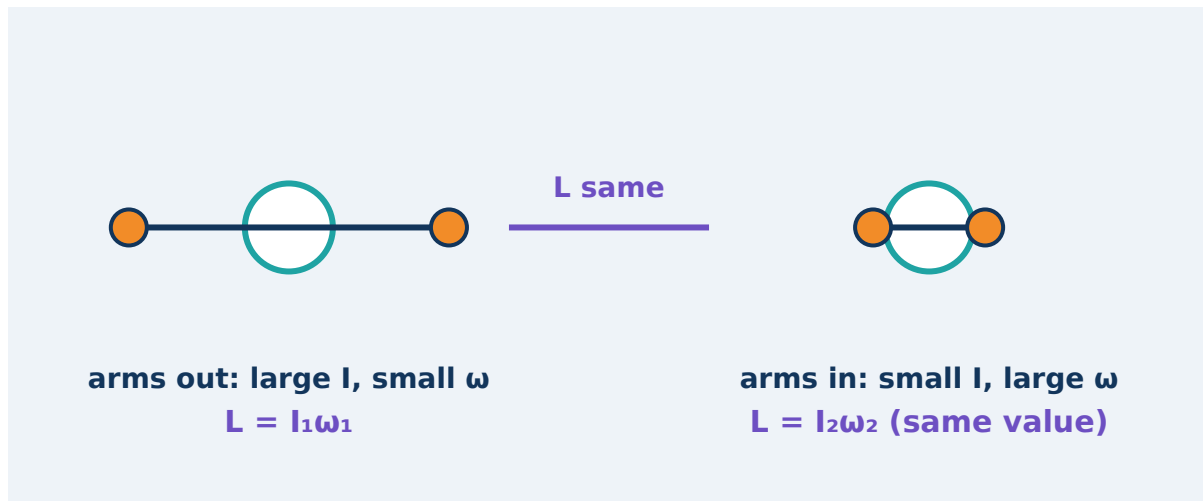
Angular momentum about a point is the vector $L = r \times p$. For a rigid body rotating about a fixed axis, $L = I\omega$ in $\text{kg m}^2/\text{s}$. The net external torque equals its rate of change: $\Sigma\tau = dL/dt$.

The value of L depends on the point chosen, because r is measured from that point. A cricket ball flying past the umpire has angular momentum about the umpire even though it never moves in a circle.

When is angular momentum conserved?

Key law

If the net external torque on a system is zero, $dL/dt = 0$ and the total angular momentum stays constant: $L_{\text{initial}} = L_{\text{final}}$. Internal forces cannot change it.



Pulling mass inward cuts I , so ω must rise to keep $L = I\omega$ constant.

A skater in a spin has no external torque about the vertical axis (friction with the ice is tiny), so $L\omega$ is fixed. Pulling the arms in lowers I and raises ω . The kinetic energy $K = L^2/2I$ rises too, because the skater's muscles do positive work on the arms as they move inward.

Worked example 1

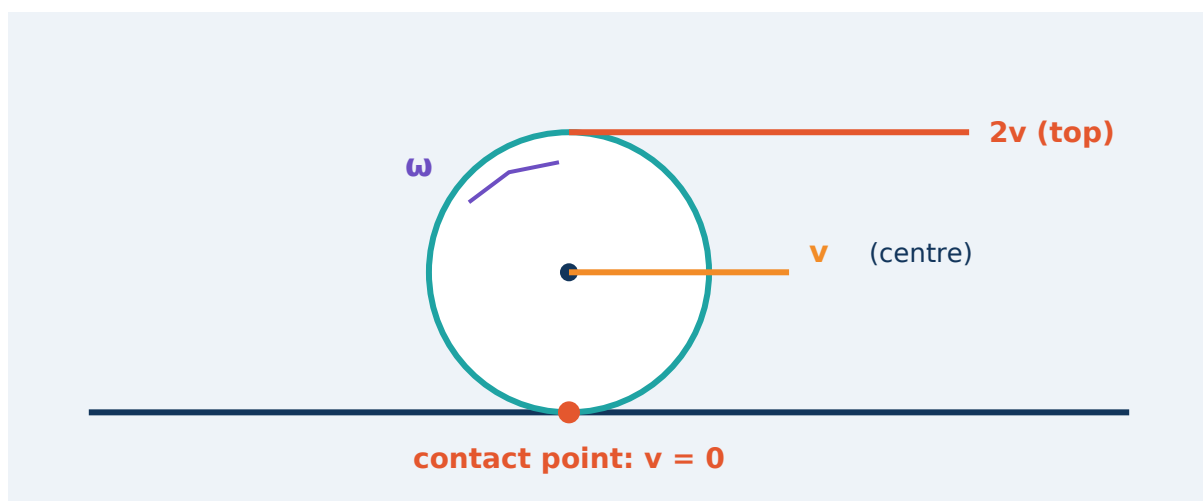
A skater spins at $\omega_1 = 4.0$ rad/s with $I_1 = 3.2$ kg m². She pulls her arms in so that $I_2 = 1.6$ kg m². Conservation gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (3.2 \times 4.0) / 1.6 = \mathbf{8.0}$ rad/s. Kinetic energy rises from $\frac{1}{2}(3.2)(4.0)^2 = 25.6$ J to $\frac{1}{2}(1.6)(8.0)^2 = 51.2$ J, so she does **25.6 J** of work.

What does rolling without slipping mean?

When a wheel rolls without slipping, the contact point is instantaneously at rest relative to the ground. Differentiating the arc length $s = R\theta$ with respect to time gives the two rolling conditions below.

Formula

$v_{\text{cm}} = R\omega$ and $a_{\text{cm}} = R\alpha$. Here v_{cm} is the speed of the centre in m/s, R the radius in m, ω the angular speed in rad/s, a_{cm} the acceleration of the centre in m/s² and α the angular acceleration in rad/s². Total kinetic energy: $K = \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I \omega^2$.



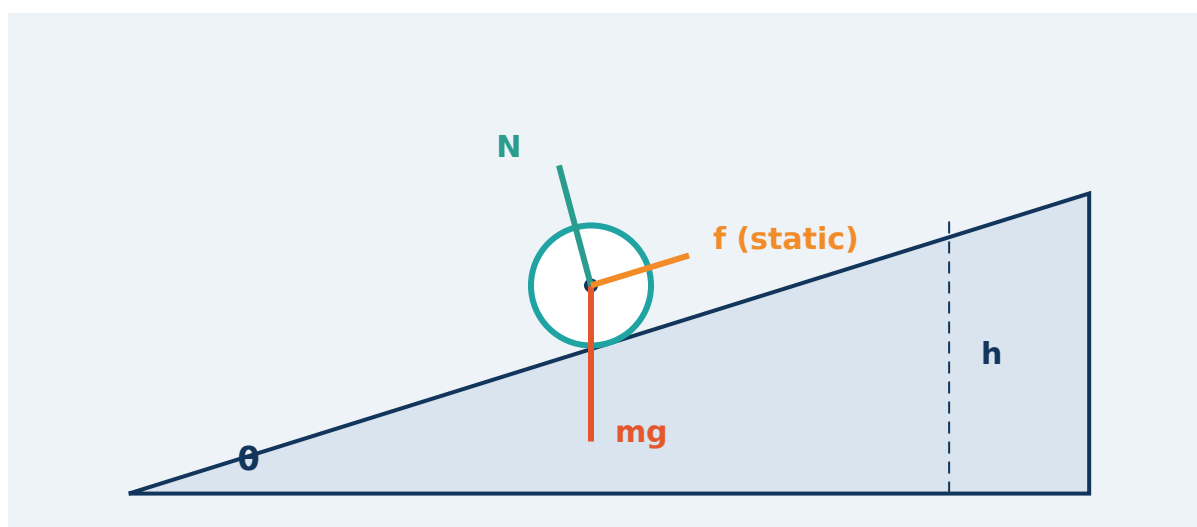
Rolling without slipping: every point is a mix of translation at v and rotation about the centre.

Because the contact point does not slide, static friction does no work on a rolling object, and mechanical energy is conserved when no other dissipative force acts.

How fast does an object roll down a slope?

Write $I = \beta m R^2$. The rolling condition turns the rotational term into $\frac{1}{2} \beta m v^2$, so $K = \frac{1}{2} m v^2 (1 + \beta)$. Energy conservation from rest through a height drop h gives $v = \sqrt{(2gh / (1 + \beta))}$.

Newton's second law along the slope together with the torque equation about the centre gives $a = g \sin \theta / (1 + \beta)$. Mass and radius cancel, so only the shape matters.



Forces on a disc rolling down a slope: static friction supplies the torque but does no work.

Shape (mass m , radius R)	β in $I = \beta m R^2$	Fraction of K that is rotational	a down slope
Hoop or thin ring	1	$1/2$	$0.500 g \sin \theta$
Thin spherical shell	$2/3$	$2/5$	$0.600 g \sin \theta$
Solid disc or cylinder	$1/2$	$1/3$	$0.667 g \sin \theta$
Solid sphere	$2/5$	$2/7$	$0.714 g \sin \theta$

Worked example 2

A solid disc ($\beta = 0.5$) rolls from rest down a ramp that drops 0.60 m. Then $v = \sqrt{(2 \times 9.81 \times 0.60 / 1.5)} = \sqrt{7.848} = \mathbf{2.8 \text{ m/s}}$. For a disc of radius 0.15 m, $\omega = v/R = 2.80 / 0.15 = \mathbf{19 \text{ rad/s}}$. A hoop released from the same ramp reaches only $\sqrt{(9.81 \times 0.60)} = 2.4 \text{ m/s}$.

What friction does rolling need?

The friction force along the slope is $f = \beta m g \sin \theta / (1 + \beta)$. It must not exceed $\mu_s m g \cos \theta$, so rolling without slipping needs $\mu_s \geq \beta \tan \theta / (1 + \beta)$. A steeper slope or a hoop demands more friction. On very smooth ice the same object would slide instead.

Common mistake

Do not say that friction slows a rolling wheel by doing negative work. Static friction acts at a point that is not moving, so it does no work. It only converts some of the gravitational energy into rotation instead of translation.

Lina's tip

Choose your pivot to kill unknown forces. Taking torques and angular momentum about the contact point or the fixed axis removes friction and normal forces from the equation, leaving only the quantities you know.

Key points to remember

- Rigid body: $L = I\omega$ (kg m²/s); particle: $L = m v r \sin\theta$; $\Sigma\tau = dL/dt$.
- If $\Sigma\tau_{\text{ext}} = 0$, then $I_1\omega_1 = I_2\omega_2$ and L is conserved.
- Rolling without slipping: $v_{\text{cm}} = R\omega$, $a_{\text{cm}} = R\alpha$, and $K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$.
- Down a slope: $a = g \sin\theta / (1 + \beta)$ for $I = \beta m R^2$; shape matters, mass and radius do not.
- Static friction on a rolling object does no work.

Angular momentum and rolling motion: frequently asked questions

What is the difference between torque and angular momentum?

Torque is the rotational effect of a force and is measured in N m. Angular momentum is the quantity of rotation stored in a system, measured in kg m²/s. Torque is the rate of change of angular momentum, $\Sigma\tau = dL/dt$.

Why does a solid cylinder beat a hollow one down a slope?

Both lose the same gravitational energy, but the hollow cylinder has more of its mass at the rim, so a larger share of the energy goes into spinning. That leaves less for translation, so it moves more slowly.

Is angular momentum conserved in a collision?

Yes, whenever the net external torque about the chosen point is zero, even if kinetic energy is lost. A clay lump landing on a turntable is a typical case: L is conserved and K decreases.

Does friction act on a wheel that rolls at constant speed on a flat road?

No friction is needed for constant speed. Static friction appears only when the wheel accelerates or on a slope, and it never does work as long as the wheel does not slip.

How do you find the speed of the top of a rolling wheel?

Add the translational speed v of the centre to the tangential speed $R\omega = v$ from the rotation. The top moves at $2v$ and the contact point at 0 relative to the ground.