

Angular momentum and rolling motion

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit in bold. Where energy is lost, say where it goes. Compare your reasoning with the method, not only the number, because the free-response section awards marks for the physical argument.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 6

Problem 1 Rolling bicycle wheel

- $\omega = v/R = 5.1 / 0.34 = \mathbf{15 \text{ rad/s}}$.
- $f = \omega/2\pi = 15 / 6.283 = \mathbf{2.4 \text{ rev/s}}$.
- The top point moves at $v + R\omega = 2v = \mathbf{10.2 \text{ m/s}}$.

Problem 2 Cricket ball and the umpire

- $L = m v r_{\perp} = 0.16 \times 30 \times 1.5 = \mathbf{7.2 \text{ kg m}^2/\text{s}}$.
- $L = 0.16 \times 30 \times 0.50 = \mathbf{2.4 \text{ kg m}^2/\text{s}}$.
- $L = r \times p$ depends on the lever arm measured from the chosen point, so a point closer to the line gives a smaller value; L is zero for any point on the line itself.

Problem 3 Spinning disc

- $I = \frac{1}{2} M R^2 = \frac{1}{2} (3.0)(0.20)^2 = \mathbf{0.060 \text{ kg m}^2}$.
- $L = I\omega = 0.060 \times 12 = \mathbf{0.72 \text{ kg m}^2/\text{s}}$.
- $K = \frac{1}{2} I \omega^2 = \frac{1}{2} (0.060)(144) = \mathbf{4.3 \text{ J}}$.

Problem 4 Skater pulling in her arms

- $I_1 \omega_1 = 0.40 I_1 \omega_2$, so $\omega_2 = 2.0 / 0.40 = \mathbf{5.0 \text{ rad/s}}$.
- $K \propto I\omega^2$, so $K_2/K_1 = 0.40 \times (5.0/2.0)^2 = 0.40 \times 6.25 = \mathbf{2.5}$. Her muscles supply the extra energy.

Problem 5 Order of the race

- $a = g \sin \theta / (1 + \beta)$ and β is $2/5$, $1/2$, $2/3$, 1 . The order is **solid sphere, solid disc, thin shell, hoop**.
- In $a = g \sin \theta / (1 + \beta)$ both m and R cancel, because $I = \beta m R^2$ scales with both; only the mass distribution (β) matters.

Problem 6 Energy split

Rotational fraction = $\beta / (1 + \beta)$.

1. $1/2 = \mathbf{50\%}$.
2. $0.4 / 1.4 = 0.286 = \mathbf{29\%}$.

Problem 7 Sphere down a ramp

1. $v = \sqrt{2gh/(1 + \beta)} = \sqrt{(2 \times 9.81 \times 0.35) / 1.4} = \sqrt{4.905} = \mathbf{2.2\text{ m/s}}$.
2. $\omega = v/R = 2.215 / 0.040 = \mathbf{55\text{ rad/s}}$.
3. $K = mgh = 0.50 \times 9.81 \times 0.35 = \mathbf{1.7\text{ J}}$.

Problem 8 Hoop on a 25° slope

1. $a = g \sin \theta / 2 = 9.81 \times 0.4226 / 2 = \mathbf{2.07\text{ m/s}^2}$.
2. $t = \sqrt{(2 \times 3.0) / 2.073} = \sqrt{2.894} = \mathbf{1.7\text{ s}}$.
3. $v = a t = 2.073 \times 1.701 = \mathbf{3.5\text{ m/s}}$ (check: $v^2 = 2aL = 12.4$).

Problem 9 Friction on a cylinder

1. $f = (\beta/(1 + \beta)) m g \sin \theta = (1/3)(4.0)(9.81)(0.500) = \mathbf{6.5\text{ N}}$.
2. $N = mg \cos \theta = 4.0 \times 9.81 \times 0.866 = 34.0\text{ N}$, so $\mu_s \geq 6.54 / 34.0 = \mathbf{0.19}$.

Problem 10 Clay on a turntable

1. $I_{\text{clay}} = m r^2 = 0.50 \times 0.16 = 0.080\text{ kg m}^2$. L conserved: $0.80 \times 6.0 = 0.88 \omega$, so $\omega = \mathbf{5.5\text{ rad/s}}$.
2. $K_1 = \frac{1}{2}(0.80)(36) = 14.4\text{ J}$; $K_2 = \frac{1}{2}(0.88)(5.45)^2 = 13.1\text{ J}$. Lost: $\mathbf{1.3\text{ J}}$, turned into heat in the sticking process.

Problem 11 Child on a roundabout

1. $I_1 = 250 + 40 \times 2.0^2 = 410\text{ kg m}^2$; $L = 410 \times 0.80 = \mathbf{328\text{ kg m}^2/\text{s}}$.
2. $I_2 = 250 + 40 \times 0.50^2 = 260\text{ kg m}^2$; $\omega_2 = 328 / 260 = \mathbf{1.3\text{ rad/s}}$.

Problem 12 Bullet and rod

1. $L = m v \ell = 0.020 \times 300 \times 0.80 = \mathbf{4.8\text{ kg m}^2/\text{s}}$.
2. $I = \frac{1}{3}(1.2)(0.80)^2 + 0.020(0.80)^2 = 0.256 + 0.0128 = \mathbf{0.269\text{ kg m}^2}$.
3. $\omega = L/I = 4.8 / 0.2688 = \mathbf{18\text{ rad/s}}$.

Problem 13 Reading an L-t graph

1. $\Sigma \tau = dL/dt = (14.0 - 2.0) / 4.0 = \mathbf{3.0\text{ N m}}$.
2. $\alpha = \tau/I = 3.0 / 1.5 = \mathbf{2.0\text{ rad/s}^2}$.
3. The area under a torque-time graph is **the change in angular momentum** (the angular

impulse).

Problem 14 Spot the error

1. No friction force is needed at constant speed, and static friction acts at a point at rest, so it does no work even when present.
2. **A ball rolling without slipping at constant speed keeps its kinetic energy; no net force acts on it, so it does not slow down** (apart from small rolling resistance).

Problem 15 Angular momentum about the contact point

1. $\omega = 4.0/0.30 = 13.3 \text{ rad/s}$; $I = \frac{1}{2}(2.0)(0.09) = 0.090 \text{ kg m}^2$; $L_{\text{cm}} = 0.090 \times 13.3 = \mathbf{1.2 \text{ kg m}^2/\text{s}}$.
2. Add $m v R = 2.0 \times 4.0 \times 0.30 = 2.4$, so $L = 1.2 + 2.4 = \mathbf{3.6 \text{ kg m}^2/\text{s}}$ (equal to $(3/2) m v R$).

Problem 16 Hoop against disc

1. Hoop: $v = \sqrt{(2 \times 9.81 \times 2.0 / 2)} = \mathbf{4.4 \text{ m/s}}$. Disc: $v = \sqrt{(2 \times 9.81 \times 2.0 / 1.5)} = \mathbf{5.1 \text{ m/s}}$.
2. $a_{\text{hoop}} / a_{\text{disc}} = (1/2)/(2/3) = 0.75$, and both start at rest at the same moment, so the hoop has covered $0.75 \times 5.0 = \mathbf{3.75 \text{ m}}$, which is 1.25 m behind.

Problem 17 Sphere on a loop-the-loop

1. At the top the weight alone supplies the centripetal force: $m g = m v^2/R$, so $v^2 = gR$.
2. Energy: $m g h = m g (2R) + \frac{1}{2} m v^2(1 + 0.4) = 2mgR + 0.7 m g R$, so $h = 2.7 R = \mathbf{1.35 \text{ m}}$.

Problem 18 Falling yo-yo

1. $m g - T = m a$ and $T R = I \alpha = \frac{1}{2} m R^2 (a/R)$, so $T = \frac{1}{2} m a$. Then $a = g/(1 + \frac{1}{2}) = \mathbf{6.5 \text{ m/s}^2}$.
2. $T = \frac{1}{2} m a = \frac{1}{2} (0.20)(6.54) = \mathbf{0.65 \text{ N}}$.
3. $v = \sqrt{(2 a s)} = \sqrt{(2 \times 6.54 \times 1.5)} = \mathbf{4.4 \text{ m/s}}$.

Problem 19 Child jumping on a roundabout

1. $L = m v R = 35 \times 4.0 \times 2.5 = 350 \text{ kg m}^2/\text{s}$. $I = 300 + 35 \times 2.5^2 = 518.75 \text{ kg m}^2$. $\omega = 350 / 518.75 = \mathbf{0.67 \text{ rad/s}}$.
2. $K_1 = \frac{1}{2}(35)(16) = 280 \text{ J}$; $K_2 = \frac{1}{2}(518.75)(0.6747)^2 = 118 \text{ J}$. Lost: **162 J**, dissipated as the child's feet grip and slide.

Problem 20 Torque from calculus

1. $L = \int \tau dt = \int 6.0t dt = \mathbf{3.0t^2 \text{ kg m}^2/\text{s}}$.
2. At $t = 4.0 \text{ s}$, $L = 3.0 \times 16 = 48 \text{ kg m}^2/\text{s}$, so $\omega = L/I = 48 / 2.0 = \mathbf{24 \text{ rad/s}}$. The torque is τ

$= 6.0 \times 4.0 = 24 \text{ N m}$, so $\alpha = \tau/I = \mathbf{12 \text{ rad/s}^2}$.

3. $W = \Delta K = L^2/2I = 48^2 / 4.0 = \mathbf{576 \text{ J}}$. Check: $\int \tau \omega \, dt = \int 9t^3 \, dt = 9 \times 256 / 4 = 576 \text{ J}$.