

Atomic energy levels and line spectra

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer shows the formula, the substitution and the final result with its unit. Check your method as well as your number: most marks in an examination go to showing the energy gap first, then converting it to a wavelength.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB E.1

Problem 1 Energy levels in words

1. An energy level is one of the discrete energies that a bound electron may have.
2. The allowed energies are fixed by the structure of the atom, so no state exists between the levels; the electron can only jump from one level to another.
3. **The ground state is the lowest energy level**, and the electron must gain energy to leave it.

Problem 2 Electronvolts to joules

$$E = 4.5 \times 1.60 \times 10^{-19} = \mathbf{7.2 \times 10^{-19} \text{ J.}}$$

Problem 3 Joules to electronvolts

$$E = 3.84 \times 10^{-19} / 1.60 \times 10^{-19} = \mathbf{2.40 \text{ eV.}}$$

Problem 4 Wavelength of a 2.0 eV photon

$\lambda = 1240 / 2.0 = \mathbf{620 \text{ nm}}$. This lies between 400 nm and 700 nm, so it is visible (orange light).

Problem 5 Reading a level diagram

1. Ground state -7.2 eV up to 0 eV : **7.2 eV**.
2. The routes are $3 \rightarrow 2$, $2 \rightarrow 1$ and $3 \rightarrow 1$, so there are **three** photon energies.
3. $3 \rightarrow 2$: 1.8 eV , $\lambda = 1240 / 1.8 = 689 \text{ nm}$. $2 \rightarrow 1$: 3.6 eV , $\lambda = 344 \text{ nm}$. $3 \rightarrow 1$: 5.4 eV , $\lambda = 230 \text{ nm}$.

Only 1.8 eV (689 nm, red) is visible; the other two are ultraviolet.

Problem 6 The $n = 4$ to $n = 2$ line

$$\Delta E = 3.40 - 0.85 = 2.55 \text{ eV. } \lambda = 1240 / 2.55 = \mathbf{486 \text{ nm}} \text{ (blue-green).}$$

Problem 7 Ionising from $n = 2$

Energy needed = $0 - (-3.40) = 3.40$ eV. $\lambda = 1240 / 3.40 = \mathbf{365\text{ nm}}$. This is below 400 nm, so it is ultraviolet.

Problem 8 Photons from a sodium lamp

1. $E = hc / \lambda = (6.63 \times 10^{-34} \times 3.00 \times 10^8) / (589 \times 10^{-9}) = 3.38 \times 10^{-19}$ J. In eV: $3.38 \times 10^{-19} / 1.60 \times 10^{-19} = \mathbf{2.11\text{ eV}}$.
2. $N = P / E = 12 / 3.38 \times 10^{-19} = \mathbf{3.55 \times 10^{19}\text{ photons per second}}$.

Problem 9 Frequency of red hydrogen light

$$f = c / \lambda = 3.00 \times 10^8 / (656 \times 10^{-9}) = \mathbf{4.57 \times 10^{14}\text{ Hz}}$$

Problem 10 Identifying the line

1. Highest energy means shortest wavelength: **line A** (410 nm).
2. $E = 1240 / 410 = \mathbf{3.02\text{ eV}}$.
3. The gap is $3.40 - 0.38 = 3.02$ eV, equal to the photon energy found in part 2, so the line is from $n = 6 \rightarrow n = 2$.

Problem 11 Dark lines from cool gas

Atoms in the cool gas absorb photons whose energy equals the gap between two of their levels. These are the same gaps that give emission when the gas is hot. The absorbed photons are re-emitted in random directions, so the intensity at those wavelengths is much lower in the forward beam and the lines look dark. **The dark lines are at the same wavelengths because both processes involve the same energy gaps.**

Problem 12 Filament lamp or neon sign

The filament is a hot solid and gives a continuous spectrum, with every colour merged smoothly. The neon sign contains low-pressure gas, whose atoms have discrete levels, so it gives a line spectrum of separate bright lines. **Continuous for the hot solid, line spectrum for the gas.**

Problem 13 Spot the errors

Mistake 1: the energy of the $n = 3$ level is -1.51 eV, not $+1.51$ eV, because bound states lie below the zero of energy. Mistake 2: to ionise from $n = 3$ the electron must be raised to 0 eV, which needs $0 - (-1.51) = 1.51$ eV, not $13.6 - 1.51$ eV. **Correct level energy -1.51 eV; ionisation energy from $n = 3$ is 1.51 eV.**

Problem 14 An electron beam in hydrogen

1. Excitation to $n = 2$ needs $13.6 - 3.40 = 10.2$ eV, which is less than 11.0 eV. Excitation to $n = 3$ needs $13.6 - 1.51 = 12.09$ eV, which is more than 11.0 eV.
2. $KE = 11.0 - 10.2 = \mathbf{0.8 \text{ eV}}$.
3. $\lambda = 1240 / 10.2 = \mathbf{122 \text{ nm}}$ (ultraviolet).

Problem 15 Routes down from $n = 4$

1. Transitions: $4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$, $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$, which is **six different transitions**.
2. The largest gap is $4 \rightarrow 1$: $13.6 - 0.85 = 12.75$ eV, $\lambda = 1240 / 12.75 = \mathbf{97.3 \text{ nm}}$.
3. The smallest gap is $4 \rightarrow 3$: $1.51 - 0.85 = 0.66$ eV, $\lambda = 1240 / 0.66 = 1880 \text{ nm} = \mathbf{1.88 \text{ }\mu\text{m}}$, in the infrared.

Problem 16 Series limit

1. $E = 1240 / 365 = \mathbf{3.40 \text{ eV}}$.
2. The series limit corresponds to an electron falling from $n = \infty$ (0 eV) to $n = 2$, so the photon energy equals the energy needed to ionise the $n = 2$ level: $E(n = 2) = -3.40$ eV.
3. $A = 3.40 \times 2^2 = 13.6$ eV, so $E(n = 1) = \mathbf{-13.6 \text{ eV}}$.

Problem 17 Reading a stellar spectrum

1. Hydrogen atoms absorb photons whose energy equals a gap between levels, so light of those wavelengths is removed from the beam.
2. $E = 1240 / 434 = \mathbf{2.86 \text{ eV}}$.
3. Final energy $= -3.40 + 2.86 = -0.543$ eV. Then $n^2 = 13.6 / 0.543 = 25.1$, so **$n = 5.0$** (the $n = 2 \rightarrow 5$ transition).

Problem 18 Levels from three lines

1. $1240 / 620 = 2.0$ eV; $1240 / 413 = 3.0$ eV; $1240 / 248 = 5.0$ eV.
2. The largest photon energy (5.0 eV) is the $3 \rightarrow 1$ transition, so the third level is $-9.0 + 5.0 = -4.0$ eV. The smallest gap (2.0 eV) lies between the upper two levels, so the middle level is $-4.0 - 2.0 = -6.0$ eV. Check: $-9.0 + 3.0 = -6.0$ eV.
3. $2.0 + 3.0 = 5.0$ eV, so the largest photon energy equals the sum of the other two. **Levels at -9.0 , -6.0 and -4.0 eV.**

Problem 19 Excitation by photons of one energy

1. Final energy $= -13.6 + 12.09 = -1.51$ eV, which is exactly the $n = 3$ level, so absorption is possible.
2. From $n = 3$: $3 \rightarrow 1$ (12.09 eV, $1240 / 12.09 = 103 \text{ nm}$); or $3 \rightarrow 2$ (1.89 eV, 656 nm) followed by $2 \rightarrow 1$ (10.2 eV, 122 nm). The wavelengths are **103 nm, 656 nm and 122 nm**.

3. Only 656 nm is visible.

Problem 20 Fluorescent coating

1. $E(254 \text{ nm}) = 1240 / 254 = \mathbf{4.88 \text{ eV}}$; $E(540 \text{ nm}) = 1240 / 540 = \mathbf{2.30 \text{ eV}}$.

2. Energy lost = $4.88 - 2.30 = \mathbf{2.58 \text{ eV}}$ (it heats the coating).

3. Efficiency = $2.30 / 4.88 \times 100 \% = \mathbf{47 \%}$.