

Motion with derivatives and integrals

AP Physics C (Mechanics and E&M, calculus-based) · Test answers (mark scheme)

Marks are shown for each step. Method marks are given for a correct derivative or integral even if the arithmetic slips, but final answers need units.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 1

Question 1 Derivatives of a position function (3 marks)

a. $v = 3t^2 - 12t + 9$, $a = 6t - 12$.

b. $3(t - 1)(t - 3) = 0$, so at rest at $t = 1 \text{ s}$ and $t = 3 \text{ s}$.

Marking: each derivative (1); both times (1).

Question 2 Integrating acceleration (2 marks)

a. $v = 2 + \int 6t \, dt = 2 + 3t^2$.

b. $v(3) = 2 + 27 = 29 \text{ m/s}$.

Marking: integral with constant (1); value (1).

Question 3 Reading a velocity graph (3 marks)

a. Area = $\frac{1}{2}(4)(8) + 6(8) + \frac{1}{2}(4)(8) = 80 \text{ m}$.

b. $a = \text{slope} = 8 / 4 = 2 \text{ m/s}^2$.

Marking: correct areas (1); total with unit (1); slope (1).

Question 4 Exponential slowing (3 marks)

a. $v = 12e^{-1} = 4.41 \text{ m/s}$.

b. $x = \int_0^\infty 12e^{-0.50t} \, dt = 12 / 0.50 = 24 \text{ m}$.

Marking: value (1); integral (1); result with unit (1).

Question 5 Acceleration as a function of position (3 marks)

a. $\int v \, dv = \int -9x \, dx$ gives $v^2 = 36 - 9x^2$. Set $v = 0$: $x^2 = 4$, so $x = 2.0 \text{ m}$.

b. $v^2 = 27$, so $v = 5.2 \text{ m/s}$.

Marking: separation and integration (1); largest x (1); speed (1).

Question 6 Why constant-acceleration equations fail (2 marks)

a. The equation is derived by integrating a constant acceleration; here a changes with time,

so it does not apply.

b. The area under an a-t graph is the **change in velocity**.

Marking: one mark each part.

Question 7 Speeding up or slowing down (2 marks)

a. At 2.5 s, $v = -2.25$ m/s and $a = +3$ m/s², opposite signs, so the speed is **decreasing**.

b. At 4.0 s, $v = 9$ m/s and $a = 12$ m/s², same sign, so the speed is **increasing**.

Marking: one mark each part.

Question 8 Simple harmonic motion (2 marks)

a. $v = 1.2 \cos(4.0t)$, so the maximum speed is **1.2 m/s**.

b. $a = -4.8 \sin(4.0t)$, so the maximum acceleration is **4.8 m/s²**.

Marking: one mark each part.