

Motion with derivatives and integrals

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These problems train you to move between position, velocity and acceleration using derivatives and integrals, and to handle accelerations that depend on time, velocity or position. They climb from direct differentiation to multi-step problems and numerical data. They suit AP Physics C mechanics unit 1 and first-year university physics.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 1

Problem 1 Velocity and acceleration from a position function (easy)

A particle has $x = 5t^2 - 3t + 1$ (m, t in s).

1. Find the velocity as a function of time.
2. Find the velocity and acceleration at $t = 2$ s.

Problem 2 Derivatives of a cubic (easy)

Aarav in Pune tracks a trolley with $x = 4 + 2t^3$.

1. Find $v(t)$ and $a(t)$.
2. Calculate v and a at $t = 3$ s.

Problem 3 Displacement from a velocity function (easy)

A cyclist has $v = 3t^2$ (m/s).

1. Find $x(t)$ if $x(0) = 0$.
2. Calculate the displacement from $t = 0$ to $t = 4$ s.
3. Calculate the average velocity over that time.

Problem 4 Integrating a simple acceleration (easy)

A rocket sled has $a = 6t$ (m/s²) and $v(0) = 2$ m/s.

1. Find $v(t)$.
2. Find v at $t = 3$ s.

Problem 5 Units of the constants (easy)

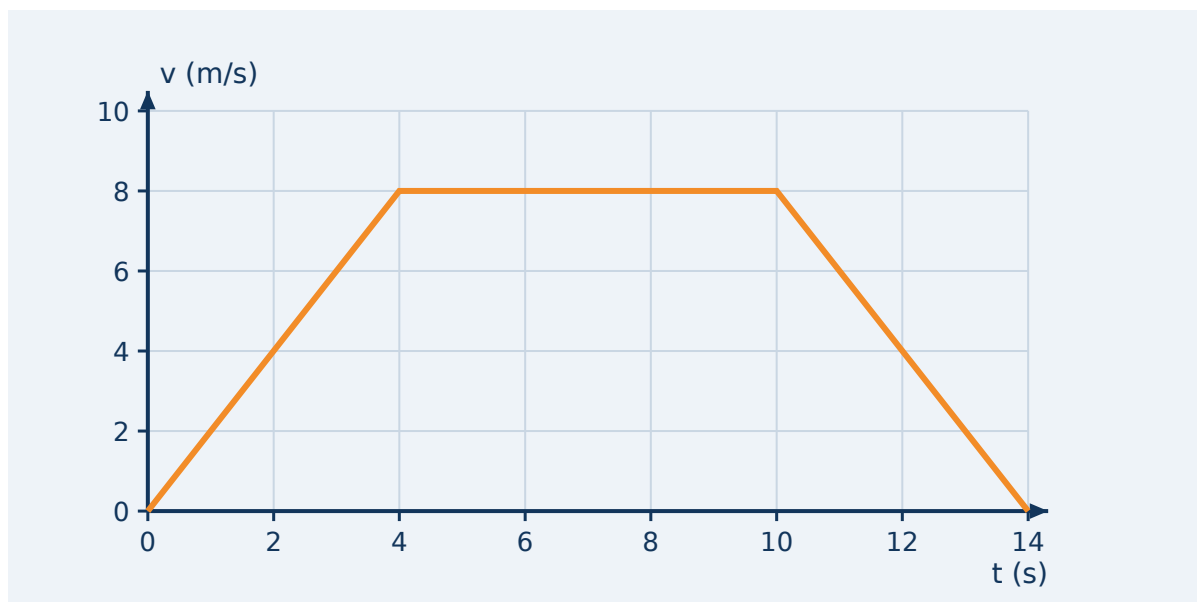
Chiamaka in Lagos models a drone with $x = (1.5 \text{ m/s}^3)t^3 + (4.0 \text{ m/s})t$.

1. Find the velocity at $t = 2.0$ s.
2. Find the acceleration at $t = 2.0$ s.

3. State the unit of each coefficient in the velocity expression.

Problem 6 Area under a v-t graph (easy)

The graph shows the velocity of a trolley.

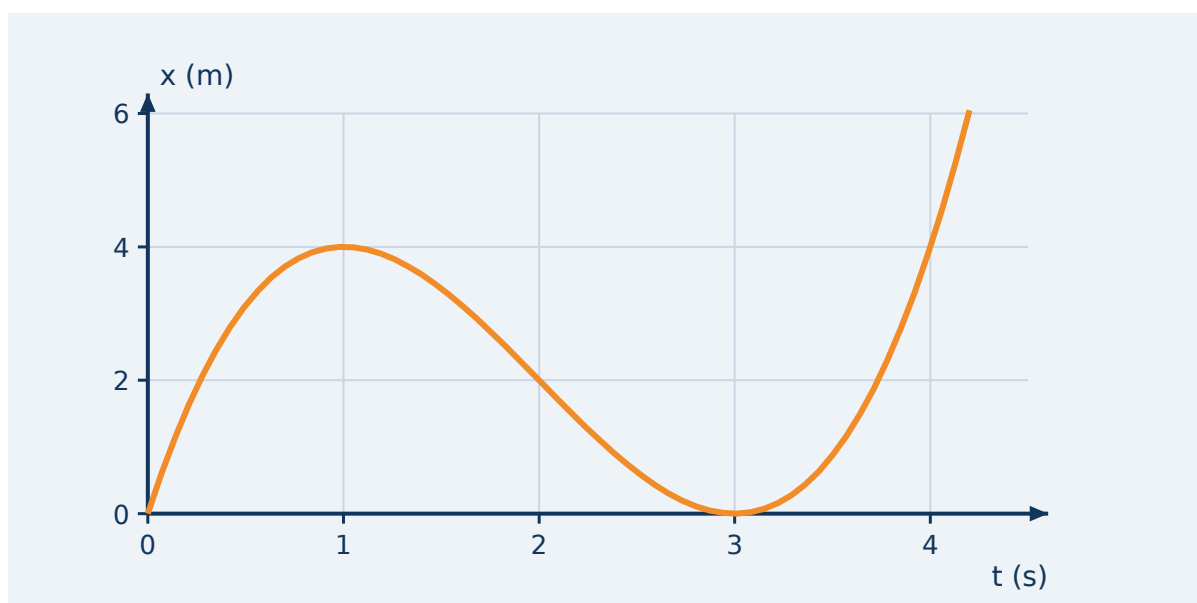


Velocity against time for a trolley.

1. Calculate the displacement from 0 to 14 s.
2. Find the acceleration during the first 4 s.
3. Find the acceleration during the last 4 s.

Problem 7 When is the particle at rest? (medium)

A particle moves along a line with $x = t^3 - 6t^2 + 9t$ (m).



Position against time for a particle on a line.

1. Find $v(t)$ and the times when the particle is at rest.
2. Find the position at each of those times.
3. Calculate the displacement and the distance travelled from 0 to 4 s.

Problem 8 Average and instantaneous velocity (medium)

A skier has $x = 2t^2$ (m).

1. Calculate the average velocity between $t = 1$ s and $t = 3$ s.
2. Calculate the instantaneous velocity at $t = 2$ s.
3. Explain why the two values are equal.

Problem 9 Two integrations (medium)

A sled has $a = 12t^2 - 4$ (m/s²), $v(0) = 1$ m/s and $x(0) = 0$.

1. Find $v(t)$ and $x(t)$.
2. Calculate v and x at $t = 2.0$ s.

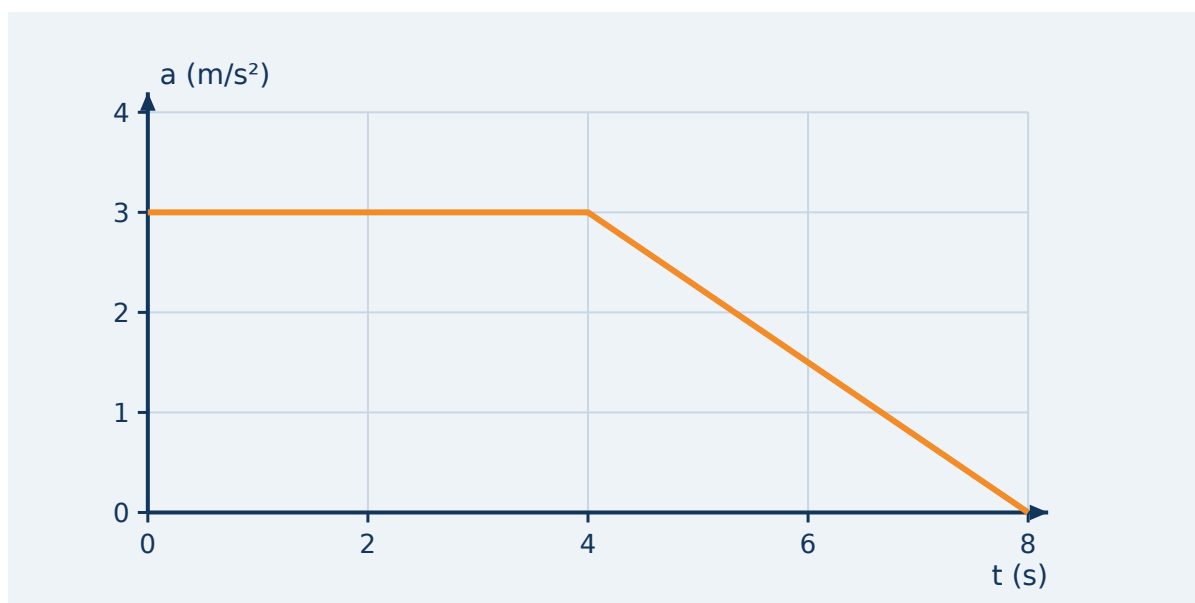
Problem 10 Thrown ball with integration (medium)

Emma in Leeds throws a ball straight up at 15 m/s. Take $a = -9.81$ m/s² and $y(0) = 0$.

1. Integrate to find $v(t)$ and $y(t)$.
2. Find the time at which the ball stops rising.
3. Calculate the maximum height.

Problem 11 Area under an a-t graph (medium)

The graph shows the acceleration of a cart that starts with $v = 2.0$ m/s.



Acceleration against time for a cart.

1. Calculate the change in velocity from 0 to 8 s.

2. Calculate the final velocity.

Problem 12 Speeding up or slowing down (medium)

A particle has $x = t^3 - 6t^2 + 9t$.

1. Find v and a at $t = 0.5$ s, 2.5 s and 4.0 s.
2. State at each time whether the speed is increasing or decreasing.

Problem 13 Exponential velocity (medium)

A puck slides with $a = -bv$, where $b = 0.50 \text{ s}^{-1}$ and $v(0) = 12 \text{ m/s}$. It is known that $v = 12e^{(-0.5t)}$.

1. Calculate v at $t = 2.0$ s.
2. Verify that the formula satisfies $a = -bv$.
3. Calculate the total distance travelled before it stops.

Problem 14 Acceleration depends on position (medium)

A mass has $a = -9x$ (SI units) and $v = 6.0 \text{ m/s}$ at $x = 0$.

1. Use $a = v \, dv/dx$ to find v^2 as a function of x .
2. Find the largest value of x .
3. Calculate v at $x = 1.0$ m.

Problem 15 Braking car (medium)

Kofi in Accra brakes so that $a = -0.8t$ (m/s^2 , t in s) from $v_0 = 20 \text{ m/s}$.

1. Find $v(t)$.
2. Find the time to stop.
3. Find the stopping distance.

Problem 16 Oscillating particle (hard)

A particle has $x = 0.30 \sin(4.0t)$ (m).

1. Find $v(t)$ and $a(t)$.
2. State the maximum speed and the maximum acceleration.
3. Calculate x , v and a at $t = 0.50$ s, and verify $a = -\omega^2 x$.

Problem 17 Drag proportional to speed squared (hard)

A sphere moves so that $a = -kv^2$, with $k = 0.020 \text{ m}^{-1}$ and $v_0 = 25 \text{ m/s}$.

1. Use $a = v \, dv/dx$ to show that $v = v_0 e^{(-kx)}$.
2. Calculate v after 50 m.

Problem 18 Acceleration that fades away (hard)

A rocket car has $a = 6.0e^{(-t/4.0)}$ (m/s²) starting from rest at $x = 0$.

1. Find $v(t)$ and the limiting speed.
2. Calculate v at $t = 4.0$ s.
3. Calculate the distance covered in the first 4.0 s.

Problem 19 Catching up (hard)

Mei in Singapore starts from rest with $x_A = 2t^2$ (m). A runner B starts 18 m ahead at $x_B = 18 + 6t$ (m).

1. Find the time at which A catches B.
2. Find the position of the meeting.
3. Compare their velocities at that time.

Problem 20 Numerical integration from data (hard)

A motion sensor records the velocity of a cart.

t (s)	0	1	2	3	4
v (m/s)	0	2.1	3.8	5.0	5.6

1. Use the trapezium rule to estimate the displacement from 0 to 4 s.
2. Estimate the acceleration at $t = 2$ s using a central difference.
3. Explain why the acceleration is decreasing.