

Motion with derivatives and integrals

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the derivative or integral, the use of initial conditions and the final value with its unit. If your result is off, check the constants of integration and whether you used distance where displacement was asked.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 1

Problem 1 Velocity and acceleration from a position function

1. $v = dx/dt = 10t - 3$.
2. $v(2) = 20 - 3 = 17 \text{ m/s}$. $a = dv/dt = 10$, so $a = 10 \text{ m/s}^2$ at all times.

Problem 2 Derivatives of a cubic

1. $v = 6t^2$, $a = 12t$.
2. $v(3) = 6 \times 9 = 54 \text{ m/s}$, $a(3) = 12 \times 3 = 36 \text{ m/s}^2$.

Problem 3 Displacement from a velocity function

1. $x = \int 3t^2 dt = t^3$.
2. $x(4) - x(0) = 64 \text{ m}$.
3. $v_{\text{avg}} = 64 / 4 = 16 \text{ m/s}$.

Problem 4 Integrating a simple acceleration

1. $v = 2 + \int 6t dt = 2 + 3t^2$.
2. $v(3) = 2 + 27 = 29 \text{ m/s}$.

Problem 5 Units of the constants

1. $v = 4.5t^2 + 4.0 = 4.5(4.0) + 4.0 = 22 \text{ m/s}$.
2. $a = 9.0t = 18 \text{ m/s}^2$.
3. 4.5 is in m/s^3 and 4.0 is in m/s , so v ends in m/s .

Problem 6 Area under a v-t graph

1. $\text{Area} = \frac{1}{2}(4)(8) + (6)(8) + \frac{1}{2}(4)(8) = 16 + 48 + 16 = 80 \text{ m}$.
2. $a = 8 / 4 = 2 \text{ m/s}^2$.
3. $a = -8 / 4 = -2 \text{ m/s}^2$.

Problem 7 When is the particle at rest?

- $v = 3t^2 - 12t + 9 = 3(t - 1)(t - 3)$, so it is at rest at **$t = 1$ s and $t = 3$ s.**
- $x(1) = 4$ m and $x(3) = 0$ m.
- $x(4) = 64 - 96 + 36 = 4$ m, so displacement = **4 m**. The path is $0 \rightarrow 4 \rightarrow 0 \rightarrow 4$ m, so distance = $4 + 4 + 4 =$ **12 m**.

Problem 8 Average and instantaneous velocity

- $(18 - 2) / 2 =$ **8 m/s**.
- $v = 4t =$ **8 m/s** at $t = 2$ s.
- For a uniformly accelerating motion v is linear in t , so the average velocity over an interval equals the instantaneous velocity at the midpoint.

Problem 9 Two integrations

- $v = 1 + \int (12t^2 - 4) dt =$ **$4t^3 - 4t + 1$** ; $x = \int v dt =$ **$t^4 - 2t^2 + t$** .
- $v(2) = 32 - 8 + 1 =$ **25 m/s**; $x(2) = 16 - 8 + 2 =$ **10 m**.

Problem 10 Thrown ball with integration

- $v = 15 - 9.81t$; $y =$ **$15t - 4.905t^2$** .
- $v = 0$ at $t = 15 / 9.81 =$ **1.53 s**.
- $y = 15(1.529) - 4.905(1.529)^2 =$ **11.5 m**.

Problem 11 Area under an a-t graph

- Area = $3 \times 4 + \frac{1}{2}(4)(3) = 12 + 6 =$ **18 m/s**.
- $v = 2.0 + 18 =$ **20 m/s**.

Problem 12 Speeding up or slowing down

- $v = 3t^2 - 12t + 9$ and $a = 6t - 12$. At 0.5 s: $v = 3.75$ m/s, $a = -9$ m/s². At 2.5 s: $v = -2.25$ m/s, $a = +3$ m/s². At 4.0 s: $v = 9$ m/s, $a = 12$ m/s².
- Opposite signs at 0.5 s and 2.5 s, so **slowing**; same signs at 4.0 s, so **speeding up**.

Problem 13 Exponential velocity

- $v = 12e^{-1} =$ **4.41 m/s**.
- $a = dv/dt = -0.5 \times 12e^{(-0.5t)} = -0.5v$. ✓
- $x = \int_0^\infty 12e^{(-0.5t)} dt = 12 / 0.5 =$ **24 m**.

Problem 14 Acceleration depends on position

- $\int v dv = \int -9x dx$ gives $v^2/2 = -4.5x^2 + 18$, so **$v^2 = 36 - 9x^2$** .
- $v = 0$ when $x^2 = 4$, so **$x = 2.0$ m**.

3. $v^2 = 36 - 9 = 27$, so $v = 5.2 \text{ m/s}$.

Problem 15 Braking car

1. $v = 20 - 0.4t^2$.
2. $20 = 0.4t^2$, so $t = \sqrt{50} = 7.07 \text{ s}$.
3. $x = 20t - (0.4/3)t^3 = 141.4 - 47.1 = 94.3 \text{ m}$.

Problem 16 Oscillating particle

1. $v = 1.2 \cos(4.0t)$; $a = -4.8 \sin(4.0t)$.
2. Maxima: **1.2 m/s** and **4.8 m/s²**.
3. At $t = 0.50$, $4.0t = 2.0 \text{ rad}$: $x = 0.30(0.909) = 0.273 \text{ m}$; $v = 1.2(-0.416) = -0.499 \text{ m/s}$; $a = -4.8(0.909) = -4.36 \text{ m/s}^2$. Check: $-16 \times 0.273 = -4.37 \text{ m/s}^2 \checkmark$. **$a = -\omega^2 x$** .

Problem 17 Drag proportional to speed squared

1. $v \, dv/dx = -kv^2$ gives $dv/v = -k \, dx$, so $\ln v = -kx + C$ and with $v(0) = v_0$, **$v = v_0 e^{-kx}$** .
2. $v = 25e^{-(0.020 \times 50)} = 25e^{-1} = 9.2 \text{ m/s}$.

Problem 18 Acceleration that fades away

1. $v = \int 6.0e^{-(t/4)} \, dt = 24(1 - e^{-(t/4)})$; as $t \rightarrow \infty$ the limit is **24 m/s**.
2. $v(4) = 24(1 - 0.368) = 15.2 \text{ m/s}$.
3. $x = 24$ from 0 to 4 = $24(1.4715) = 35.3 \text{ m}$.

Problem 19 Catching up

1. $2t^2 = 18 + 6t$ gives $t^2 - 3t - 9 = 0$, so $t = (3 + \sqrt{45}) / 2 = 4.85 \text{ s}$.
2. $x = 2(4.854)^2 = 47.1 \text{ m}$.
3. $v_A = 4t = 19.4 \text{ m/s}$ against $v_B = 6 \text{ m/s}$, so A is moving **13.4 m/s faster** than B.

Problem 20 Numerical integration from data

1. Area = $\frac{1}{2}(0 + 2.1) + \frac{1}{2}(2.1 + 3.8) + \frac{1}{2}(3.8 + 5.0) + \frac{1}{2}(5.0 + 5.6) = 1.05 + 2.95 + 4.40 + 5.30 = 13.7 \text{ m}$.
2. $a \approx (5.0 - 2.1) / 2 = 1.45 \text{ m/s}^2$.
3. The velocity rises more slowly each second, so the slope of the v - t graph falls, as when a force reduces with speed.