

Capacitors, energy storage and dielectrics

AP Physics C (Mechanics and E&M, calculus-based) · Lesson

Quick answer: Capacitance is the charge stored per unit potential difference, $C = Q/\Delta V$, measured in farads. It depends on geometry: $\epsilon_0 A/d$ for parallel plates and $2\pi\epsilon_0 L/\ln(b/a)$ for coaxial cylinders. A dielectric of constant κ multiplies C by κ , and the stored energy is $U = \frac{1}{2}C\Delta V^2 = \frac{1}{2}\epsilon_0 E^2$ per unit volume.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 10

What is capacitance and how do you calculate it?

Two conductors that carry equal and opposite charge $\pm Q$ have a potential difference ΔV between them. For any fixed shape, ΔV is proportional to Q , because doubling the charge doubles the field everywhere. The constant of proportionality depends only on geometry and on the material between the conductors.

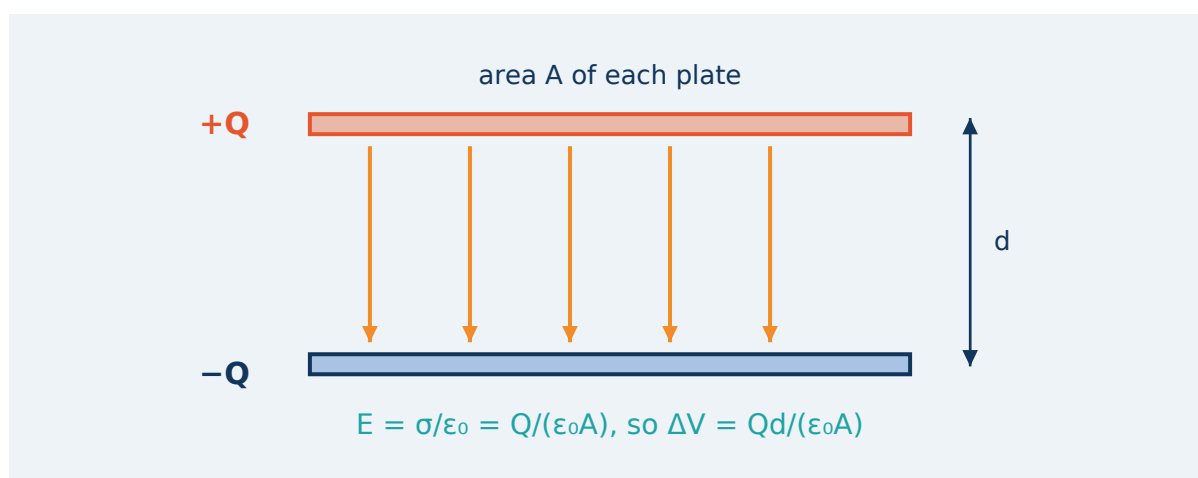
Definition

Capacitance is the charge stored per unit potential difference: $C = Q/\Delta V$. Its unit is the farad, $1 \text{ F} = 1 \text{ C/V}$.

Formula

$$C = Q/\Delta V \quad C = \epsilon_0 A/d \text{ (parallel plates)} \quad U = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = Q^2/2C \quad u = \frac{1}{2}\epsilon_0 E^2$$

Q is charge on one plate (C), ΔV is potential difference (V), A is plate area (m^2), d is gap (m), U is stored energy (J), u is energy density (J/m^3), $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.



A parallel plate capacitor: uniform field between the plates, $\Delta V = Ed$.

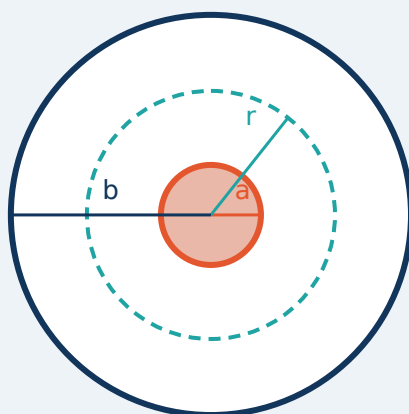
The method is always the same: put charge $\pm Q$ on the conductors, find E with Gauss's law, integrate to get $\Delta V = -\int E \cdot dl$, then divide. The geometry fixes the answer.

Worked example

Data: plates $20\text{ cm} \times 10\text{ cm}$, separated by 1.0 mm of air, connected to 12 V . **Find** C and Q .

Method: $A = 0.020\text{ m}^2$, so $C = \epsilon_0 A/d = (8.85 \times 10^{-12})(0.020)/(1.0 \times 10^{-3})$. Then $Q = C\Delta V$.

Answer: $C = 177\text{ pF}$ and $Q = 2.1\text{ nC}$.



Gauss: $E(2\pi rL) = \lambda L/\epsilon_0$

$$E = \lambda/(2\pi\epsilon_0 r)$$

$$\Delta V = (\lambda/2\pi\epsilon_0) \ln(b/a)$$

A coaxial capacitor: Gauss's law gives E , and integrating E from a to b gives the potential difference.

Worked example

Data: a coaxial cable has inner radius $a = 1.0\text{ mm}$, outer radius $b = 4.0\text{ mm}$ and length $L = 2.0\text{ m}$. **Find** C .

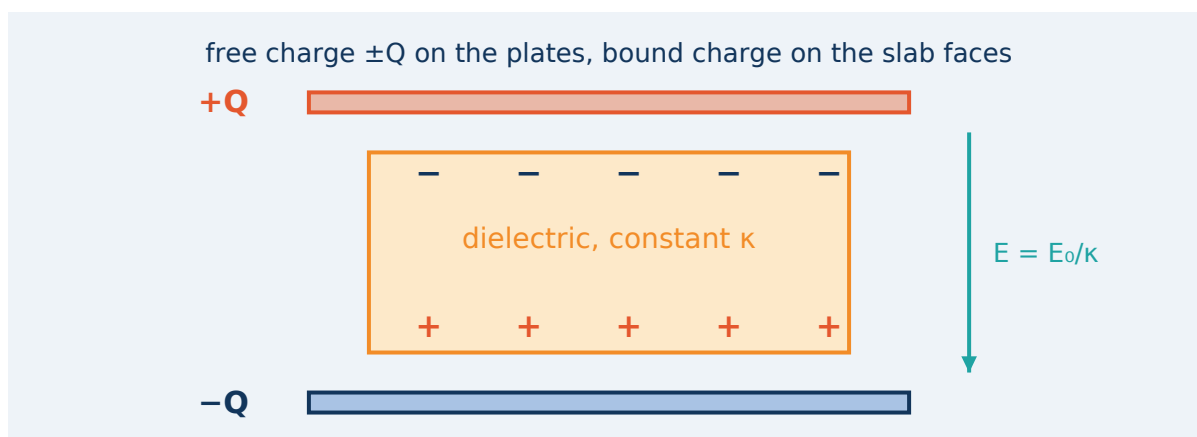
Method: with $\lambda = Q/L$, $E = \lambda/(2\pi\epsilon_0 r)$ and $\Delta V = (\lambda/2\pi\epsilon_0) \ln(b/a)$, so $C = 2\pi\epsilon_0 L/\ln(b/a)$.

Answer: $C = 80\text{ pF}$, since $2\pi\epsilon_0 L = 1.112 \times 10^{-10}\text{ F}$ and $\ln 4 = 1.386$.

Geometry	Capacitance C
Parallel plates (area A , gap d)	$\epsilon_0 A/d$
Isolated sphere, radius R	$4\pi\epsilon_0 R$
Concentric spheres, radii $a < b$	$4\pi\epsilon_0 ab/(b - a)$
Coaxial cylinders, length L	$2\pi\epsilon_0 L/\ln(b/a)$
Any of these filled with dielectric κ	κ times the empty value

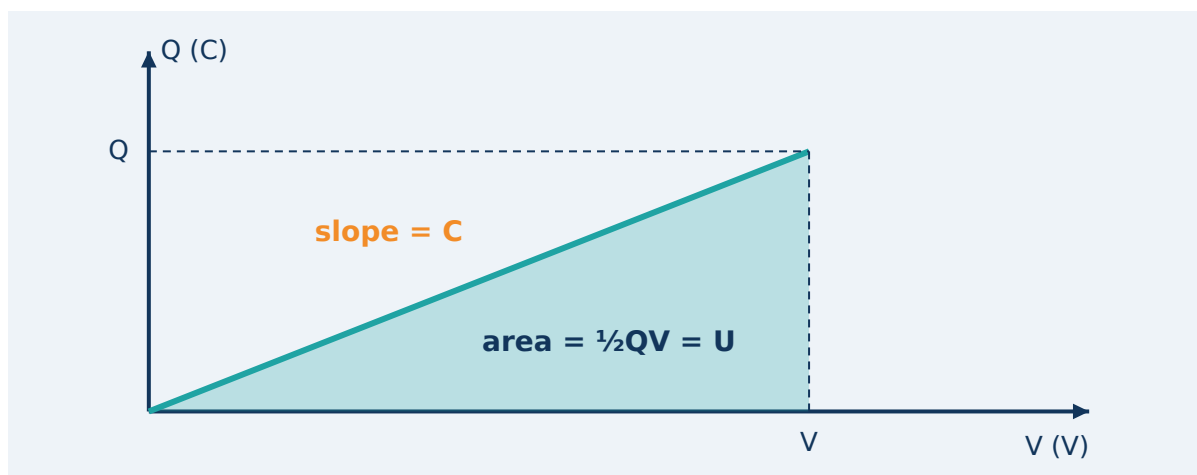
What does a dielectric do to a capacitor?

A dielectric is an insulator whose molecules polarise in a field. Its surfaces gain bound charge that opposes the field of the plates, so the net field falls to E_0/κ , where κ is the dielectric constant. For the same charge, ΔV drops by κ , so the capacitance rises by the factor κ .



Bound charge on the dielectric surfaces reduces the field between the plates.

Energy is stored in the field, not on the plates alone. Each cubic metre holds $u = \frac{1}{2}\epsilon_0 E^2$ in vacuum, and $u = \frac{1}{2}\kappa\epsilon_0 E^2$ in a dielectric. Integrating u over the volume reproduces $U = Q^2/2C$.



On a Q - V graph the slope is C and the area under the line is the stored energy.

Worked example

Data: the 177 pF capacitor is charged to 12 V ($Q = 2.12$ nC), disconnected, and then filled with a dielectric of $\kappa = 4$. **Find** the new V and the energy change.

Method: Q is fixed, so $V = Q/(\kappa C_0)$ and $U = Q^2/(2\kappa C_0) = U_0/\kappa$, with $U_0 = \frac{1}{2}C_0 V_0^2 = 12.7$ nJ.

Answer: $V = 3.0$ V and U falls from 12.7 nJ to 3.2 nJ; the slab is pulled in, doing work as it enters.

Common mistake

Do not use one rule for every case. With the battery connected, V stays fixed and Q changes. With the battery disconnected, Q stays fixed and V changes. Decide which is constant before you apply a dielectric or change d .

Lina's tip

Write down the constant quantity first, V or Q , then express every other change through $C = \kappa\epsilon_0 A/d$.

Key points to remember

- $C = Q/\Delta V$ ($1 \text{ F} = 1 \text{ C/V}$); parallel plates $C = \epsilon_0 A/d$.
- Find C by Gauss's law for E , then $\Delta V = -\int E \cdot dl$, then $C = Q/\Delta V$.
- $U = \frac{1}{2}Q\Delta V = \frac{1}{2}C\Delta V^2 = Q^2/2C$; energy density $u = \frac{1}{2}\epsilon_0 E^2$.
- Dielectric κ : $C = \kappa C_0$ and field $E = E_0/\kappa$ at the same free charge.
- Battery connected: V fixed. Battery disconnected: Q fixed.

Capacitors, energy storage and dielectrics: frequently asked questions

Does capacitance depend on charge or voltage?

No. Q and ΔV change together, but their ratio depends only on geometry and the material between the conductors. Doubling Q doubles ΔV and leaves C unchanged.

Where is the energy of a capacitor stored?

It is stored in the electric field between the conductors, with density $u = \frac{1}{2}\epsilon_0 E^2$ (or $\frac{1}{2}\kappa\epsilon_0 E^2$ in a dielectric). Integrating u over the gap gives the same result as $\frac{1}{2}C\Delta V^2$.

Why does a dielectric increase capacitance?

Polarisation puts bound charge on the surfaces that cancels part of the field. For the same free charge the field and potential difference drop, so $Q/\Delta V$ is larger by the factor κ .

What happens when a dielectric is inserted with the battery still attached?

The voltage stays fixed, so charge $Q = \kappa C_0 V$ flows onto the plates and the stored energy rises by the factor κ . The battery supplies the extra energy.

How do capacitors combine?

In parallel the capacitances add, $C = C_1 + C_2$, because the voltage is shared. In series the reciprocals add, $1/C = 1/C_1 + 1/C_2$, because the charge is shared.