

Capacitors, energy storage and dielectrics

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These twenty problems build fluency with capacitance: finding C from geometry with Gauss's law, computing stored energy and energy density, and predicting what a dielectric does when the battery is connected or removed. They run from recall to multi-step derivations in the style of AP Physics C: Electricity and Magnetism.

The set suits AP students, first-year university physics and A Level learners ready for calculus. Use $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ throughout.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 10

Problem 1 Charge on a capacitor (easy)

1. Calculate the charge stored by a $10 \mu\text{F}$ capacitor connected across 6.0 V .
2. Calculate the energy stored.

Problem 2 Parallel plate value (easy)

Emma builds a capacitor from two plates of area 0.0050 m^2 separated by 0.50 mm of air.

1. Calculate its capacitance.
2. State what happens to C if the gap is doubled.

Problem 3 Series and parallel (easy)

A $6.0 \mu\text{F}$ capacitor and a $3.0 \mu\text{F}$ capacitor are available.

1. Calculate the equivalent capacitance in series.
2. Calculate the equivalent capacitance in parallel.

Problem 4 Camera flash (easy)

A flash unit uses a $470 \mu\text{F}$ capacitor charged to 25 V .

1. Calculate the charge stored.
2. Calculate the energy stored, in joules.

Problem 5 Doubling the gap (easy)

A plate capacitor is connected to a battery and the plates are pulled to twice their separation.

1. State how C changes.
2. State how Q and the stored energy change, and explain.

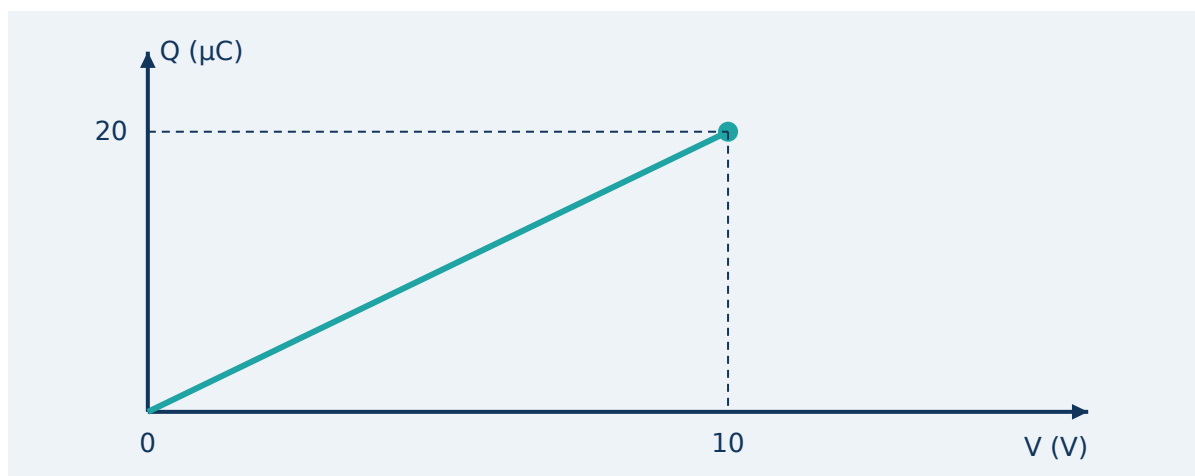
Problem 6 Dielectric constant (easy)

A 50 pF air capacitor is filled with a material of $\kappa = 3.5$.

1. Calculate the new capacitance.
2. Calculate the charge when it is connected to 9.0 V.

Problem 7 Reading a Q-V graph (easy)

The graph shows the behaviour of a capacitor.



Charge against voltage.

1. Calculate the capacitance from the slope.
2. Calculate the energy stored at 10 V from the area.

Problem 8 Field between plates (medium)

Plates of area 0.010 m^2 carry $\pm 2.0 \text{ nC}$ and are 2.0 mm apart.

1. Calculate the surface charge density.
2. Calculate the field between the plates.
3. Calculate ΔV and then $C = Q/\Delta V$.

Problem 9 Energy density (medium)

Dry air breaks down at about $3.0 \times 10^6 \text{ V/m}$. Plates of area 0.040 m^2 are 2.0 mm apart in air and the field is just at this limit.

1. Calculate the energy density.
2. Calculate the energy stored in the gap.

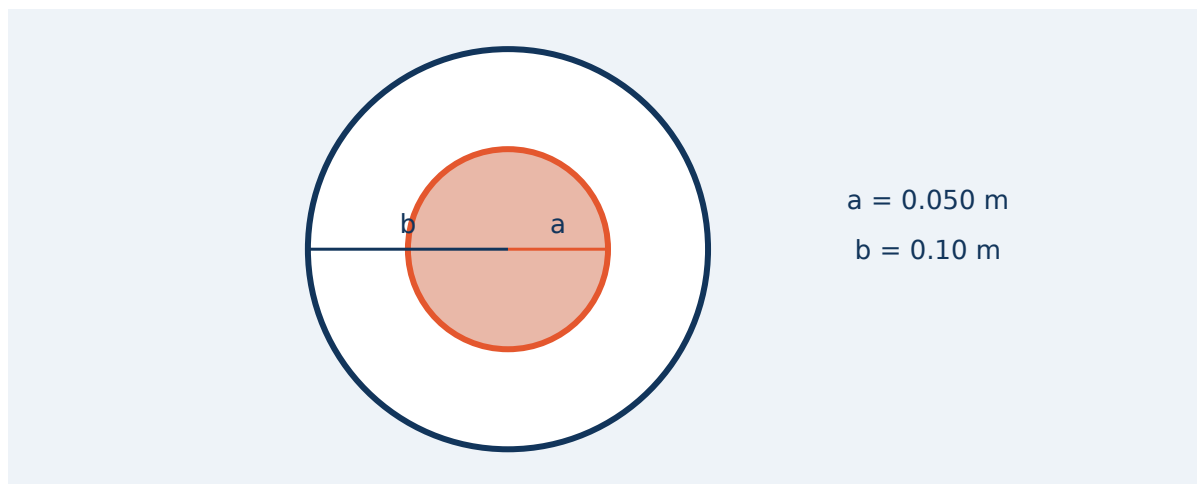
Problem 10 Coaxial cable (medium)

Bilal measures a coaxial line with inner radius 0.50 mm, outer radius 3.0 mm and length 5.0 m.

1. Use Gauss's law to show $E = \lambda/(2\pi\epsilon_0 r)$.
2. Calculate its capacitance.

Problem 11 Concentric spheres (medium)

The diagram shows two concentric conducting spheres.



Concentric spheres.

1. Show that $C = 4\pi\epsilon_0 ab/(b - a)$.
2. Calculate C for the radii shown.
3. Calculate the capacitance of an isolated sphere of radius 0.50 m.

Problem 12 Dielectric with battery (medium)

A 200 pF capacitor is held at 12 V by a battery while a slab with $\kappa = 5$ fills the gap.

1. Calculate the new capacitance.
2. Calculate the charge before and after.
3. Calculate the stored energy before and after.

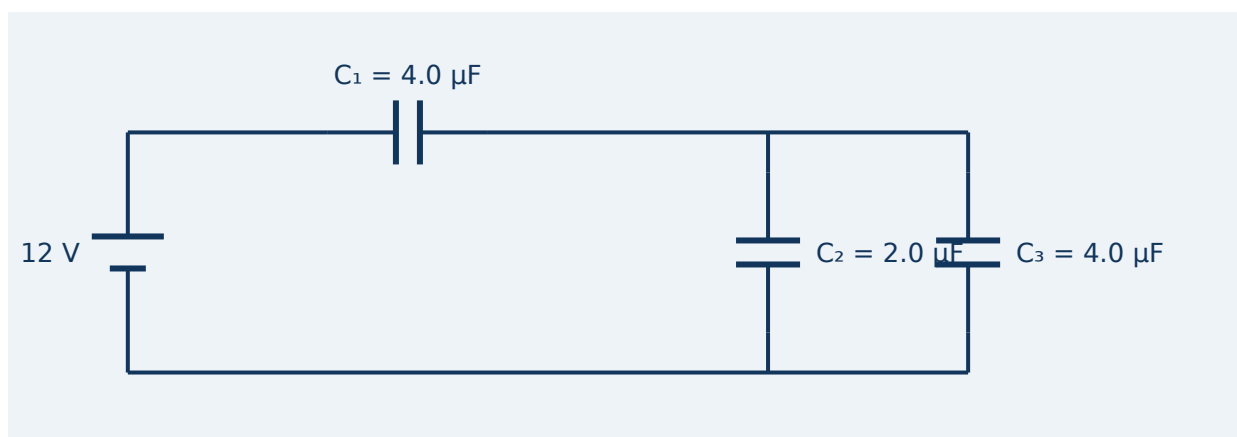
Problem 13 Dielectric, no battery (medium)

The same 200 pF capacitor is charged to 12 V, then the battery is removed before the slab is inserted.

1. State what stays constant.
2. Calculate the new voltage.
3. Calculate the new stored energy and the work done by the field in pulling the slab in.

Problem 14 Capacitor network (medium)

Study the network.



A series-parallel network across 12 V.

1. Calculate the equivalent capacitance.
2. Calculate the charge on C_1 .
3. Calculate the voltage across the parallel pair.
4. Calculate the charge on C_2 and C_3 .

Problem 15 Sharing charge (medium)

Mei charges a $6.0 \mu\text{F}$ capacitor to 10 V and then connects it, with the battery removed, to an uncharged $3.0 \mu\text{F}$ capacitor.

1. Calculate the final common voltage.
2. Calculate the energy before and after.
3. Explain where the missing energy goes.

Problem 16 Two dielectric layers (hard)

Plates of area 0.020 m^2 are separated by two layers: 1.0 mm with $\kappa = 2.0$ and 1.0 mm with $\kappa = 4.0$.

1. Treat the layers as capacitors in series and calculate C .
2. Calculate the charge at 100 V.

Problem 17 Half-filled plates (hard)

The same plates ($A = 0.020 \text{ m}^2$, $d = 2.0 \text{ mm}$) have a $\kappa = 4.0$ slab filling one half of the area, side by side.

1. Explain why the two halves are in parallel.
2. Calculate the total capacitance.

Problem 18 Work to charge (hard)

Wanjiru's capacitor has $C = 8.0 \mu\text{F}$.

1. Integrate $dW = (q/C) dq$ to show $U = Q^2/2C$.
2. Calculate U at $Q = 40 \mu\text{C}$.
3. Calculate the extra work to raise Q from $40 \mu\text{C}$ to $60 \mu\text{C}$.

Problem 19 Energy by integration (hard)

Two spherical shells have $a = 0.050$ m and $b = 0.10$ m and carry ± 2.0 nC.

1. Integrate $u = \frac{1}{2}\epsilon_0 E^2$ over the gap to show $U = (Q^2/8\pi\epsilon_0)(1/a - 1/b)$.
2. Calculate U .
3. Check using $U = Q^2/2C$.

Problem 20 Spot the error (hard)

Diego says: 'Inserting a dielectric into an isolated charged capacitor raises its capacitance, so it stores more energy.' The capacitor is 100 pF at 50 V.

1. Find the mistake.
2. Calculate the energy before and after inserting a $\kappa = 2.5$ dielectric.
3. Calculate the final voltage.