

Capacitors, energy storage and dielectrics

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit in bold. Compare your reasoning about which quantity stays fixed, because choosing the wrong constant is the most common source of errors in dielectric problems.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 10

Problem 1 Charge on a capacitor

1. $Q = C\Delta V = (10 \times 10^{-6})(6.0) = \mathbf{6.0 \times 10^{-5} \text{ C}}$ (60 μC).
2. $U = \frac{1}{2}C\Delta V^2 = \frac{1}{2}(10 \times 10^{-6})(36) = \mathbf{1.8 \times 10^{-4} \text{ J}}$.

Problem 2 Parallel plate value

1. $C = \epsilon_0 A/d = (8.85 \times 10^{-12})(0.0050)/(5.0 \times 10^{-4}) = \mathbf{8.9 \times 10^{-11} \text{ F}}$ (88.5 pF).
2. $C \propto 1/d$, so **C halves**.

Problem 3 Series and parallel

1. $1/C = 1/6 + 1/3 = 1/2$, so **C = 2.0 μF** .
2. $C = 6.0 + 3.0 = \mathbf{9.0 \mu\text{F}}$.

Problem 4 Camera flash

1. $Q = C\Delta V = (470 \times 10^{-6})(25) = \mathbf{1.2 \times 10^{-2} \text{ C}}$ (11.75 mC).
2. $U = \frac{1}{2}C\Delta V^2 = \frac{1}{2}(470 \times 10^{-6})(625) = \mathbf{0.15 \text{ J}}$.

Problem 5 Doubling the gap

1. $C = \epsilon_0 A/d$, so **C halves**.
2. With V fixed by the battery, $Q = CV$ and $U = \frac{1}{2}CV^2$ both **halve**; charge flows back to the battery.

Problem 6 Dielectric constant

1. $C = \kappa C_0 = 3.5 \times 50 = \mathbf{175 \text{ pF}}$.
2. $Q = C\Delta V = (175 \times 10^{-12})(9.0) = \mathbf{1.6 \times 10^{-9} \text{ C}}$.

Problem 7 Reading a Q-V graph

1. $C = \text{slope} = 20 \mu\text{C}/10 \text{ V} = \mathbf{2.0 \mu\text{F}}$.

2. $U = \text{area} = \frac{1}{2}(20 \times 10^{-6})(10) = \mathbf{1.0 \times 10^{-4} \text{ J}}$.

Problem 8 Field between plates

- $\sigma = Q/A = 2.0 \times 10^{-9}/0.010 = \mathbf{2.0 \times 10^{-7} \text{ C/m}^2}$.
- $E = \sigma/\epsilon_0 = 2.0 \times 10^{-7}/8.85 \times 10^{-12} = \mathbf{2.3 \times 10^4 \text{ V/m}}$.
- $\Delta V = Ed = 45 \text{ V}$, so $C = 2.0 \text{ nC}/45 \text{ V} = \mathbf{44 \text{ pF}}$, which matches $\epsilon_0 A/d$.

Problem 9 Energy density

- $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}(8.85 \times 10^{-12})(3.0 \times 10^6)^2 = \mathbf{40 \text{ J/m}^3}$ (39.8).
- Volume = $0.040 \times 0.0020 = 8.0 \times 10^{-5} \text{ m}^3$, so $U = 39.8 \times 8.0 \times 10^{-5} = \mathbf{3.2 \times 10^{-3} \text{ J}}$.

Problem 10 Coaxial cable

- Take a Gaussian cylinder of radius r and length l : $E(2\pi rl) = \lambda l/\epsilon_0$, so $E = \lambda/(2\pi\epsilon_0 r)$.
- $C = 2\pi\epsilon_0 L/\ln(b/a) = (5.56 \times 10^{-11})(5.0)/\ln 6 = 2.78 \times 10^{-10}/1.792 = \mathbf{1.6 \times 10^2 \text{ pF}}$ (155 pF).

Problem 11 Concentric spheres

- $E = kQ/r^2$ between the shells, so $\Delta V = kQ(1/a - 1/b) = kQ(b - a)/(ab)$ and $C = Q/\Delta V = ab/(4\pi\epsilon_0(b - a))$.
- $C = (1.112 \times 10^{-10})(0.050 \times 0.10)/0.050 = \mathbf{1.1 \times 10^{-11} \text{ F}}$ (11.1 pF).
- Let $b \rightarrow \infty$: $C = 4\pi\epsilon_0 R = (1.112 \times 10^{-10})(0.50) = \mathbf{56 \text{ pF}}$.

Problem 12 Dielectric with battery

- $C = \kappa C_0 = \mathbf{1000 \text{ pF}}$.
- $Q_0 = (200 \text{ pF})(12) = 2.4 \text{ nC}$ and $Q = (1000 \text{ pF})(12) = \mathbf{12 \text{ nC}}$.
- $U_0 = \frac{1}{2}(200 \times 10^{-12})(144) = 14.4 \text{ nJ}$ and $U = \mathbf{72 \text{ nJ}}$.

Problem 13 Dielectric, no battery

- The **charge $Q = 2.4 \text{ nC}$** stays constant, because there is nowhere for it to go.
- $V = Q/(\kappa C_0) = 2.4 \text{ nC}/1000 \text{ pF} = \mathbf{2.4 \text{ V}}$.
- $U = Q^2/2C = 2.88 \text{ nJ}$, a drop of $14.4 - 2.88 = \mathbf{11.5 \text{ nJ}}$ done as work pulling the slab in.

Problem 14 Capacitor network

- $C_2 + C_3 = 6.0 \text{ }\mu\text{F}$, in series with $4.0 \text{ }\mu\text{F}$: $C = (4.0 \times 6.0)/10 = \mathbf{2.4 \text{ }\mu\text{F}}$.
- $Q = CV = (2.4)(12) = \mathbf{28.8 \text{ }\mu\text{C}}$ on C_1 (the series charge).
- $V_1 = 28.8/4.0 = 7.2 \text{ V}$, so the pair has $12 - 7.2 = \mathbf{4.8 \text{ V}}$.
- $Q_2 = (2.0)(4.8) = \mathbf{9.6 \text{ }\mu\text{C}}$ and $Q_3 = (4.0)(4.8) = \mathbf{19.2 \text{ }\mu\text{C}}$.

Problem 15 Sharing charge

1. $Q = (6.0)(10) = 60 \mu\text{C}$ is conserved, so $V = 60/(6.0 + 3.0) = \mathbf{6.7 \text{ V}}$.
2. $U_0 = \frac{1}{2}(6.0 \times 10^{-6})(100) = 3.0 \times 10^{-4} \text{ J}$ and $U = \frac{1}{2}(9.0 \times 10^{-6})(6.67)^2 = 2.0 \times 10^{-4} \text{ J}$, so **one third is lost**.
3. It is dissipated as heat in the connecting wires, and partly radiated, during the transient current.

Problem 16 Two dielectric layers

1. $C_1 = \kappa_1 \epsilon_0 A/d_1 = 2(1.77 \times 10^{-13})/10^{-3} = 354 \text{ pF}$ and $C_2 = 708 \text{ pF}$; in series $C = (354 \times 708)/1062 = \mathbf{2.4 \times 10^2 \text{ pF}}$ (236 pF).
2. $Q = C\Delta V = (2.36 \times 10^{-10})(100) = \mathbf{2.4 \times 10^{-8} \text{ C}}$.

Problem 17 Half-filled plates

1. Both halves are connected to the same two plates, so each has the full voltage; charges add, which is the rule for parallel capacitors.
2. $C = \epsilon_0(A/2)/d + \kappa\epsilon_0(A/2)/d = (8.85 \times 10^{-12})(0.020)(1 + 4)/(2 \times 0.0020) = \mathbf{2.2 \times 10^2 \text{ pF}}$ (221 pF).

Problem 18 Work to charge

1. $W = \int_0^Q (q/C) dq = Q^2/2C$.
2. $U = (40 \times 10^{-6})^2/(2 \times 8.0 \times 10^{-6}) = \mathbf{1.0 \times 10^{-4} \text{ J}}$.
3. $U(60 \mu\text{C}) = 2.25 \times 10^{-4} \text{ J}$, so the extra work is $\mathbf{1.25 \times 10^{-4} \text{ J}}$.

Problem 19 Energy by integration

1. $U = \int \frac{1}{2}\epsilon_0 E^2 4\pi r^2 dr$ with $E = kQ/r^2 = (Q^2/8\pi\epsilon_0) \int$ from a to b of $dr/r^2 = \mathbf{(Q^2/8\pi\epsilon_0)(1/a - 1/b)}$.
2. $U = (4.0 \times 10^{-18}/2.224 \times 10^{-10})(10) = \mathbf{1.8 \times 10^{-7} \text{ J}}$.
3. $Q^2/2C = 4.0 \times 10^{-18}/(2 \times 1.112 \times 10^{-11}) = 1.8 \times 10^{-7} \text{ J}$, which agrees.

Problem 20 Spot the error

1. Q stays fixed when the capacitor is isolated, so $U = Q^2/2C$ **falls** as C rises.
2. $U_0 = \frac{1}{2}(100 \times 10^{-12})(2500) = 1.25 \times 10^{-7} \text{ J}$ and $U = U_0/2.5 = \mathbf{5.0 \times 10^{-8} \text{ J}}$.
3. $V = 50/2.5 = \mathbf{20 \text{ V}}$.