

# Charge storage, energy and RC time constants

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

**Quick answer:** A capacitor stores charge on two plates, and its capacitance is  $C = Q / V$  in farads. The stored energy is  $E = \frac{1}{2}CV^2$ . Capacitors in parallel add, while series capacitors combine as reciprocals. Through a resistor the voltage changes exponentially with time constant  $\tau = RC$ .

**Level:** Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.7.4

## What is a capacitor and what does capacitance mean?

A capacitor stores energy by keeping equal and opposite charges on two conducting plates separated by an insulator. The camera flash in a phone, the timer in a pedestrian crossing and the smoothing circuit in a charger all rely on this.

### Definition

**Capacitance** is the charge stored per unit potential difference:  $C = Q / V$ . It is measured in farads (F);  $1 \text{ F} = 1 \text{ C/V}$ . Most real capacitors are in microfarads ( $\mu\text{F}$ ), nanofarads (nF) or picofarads (pF).

Here  $Q$  is the magnitude of the charge on one plate, not the total charge on the capacitor, which is always zero. A capacitor of larger capacitance stores more charge at the same voltage.

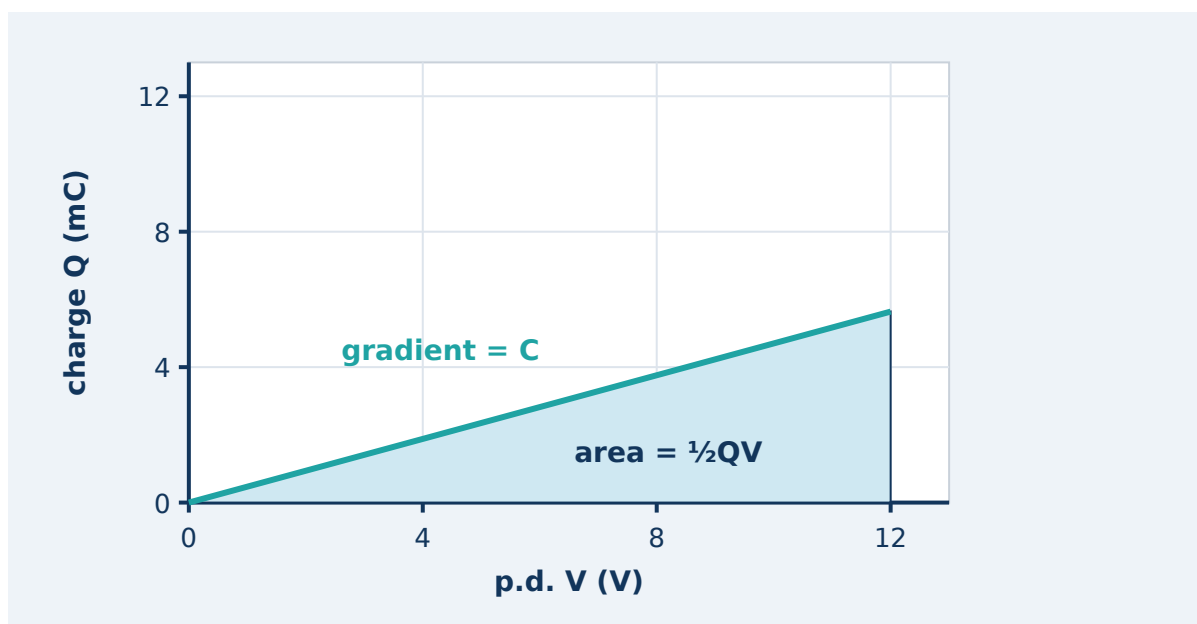
### Formula

$$Q = CV$$

$Q$  = charge on one plate (C),  $C$  = capacitance (F),  $V$  = potential difference across the capacitor (V). Unit prefixes:  $1 \mu\text{F} = 10^{-6} \text{ F}$ ,  $1 \text{ nF} = 10^{-9} \text{ F}$ ,  $1 \text{ pF} = 10^{-12} \text{ F}$ .

## How much energy does a capacitor store?

As charge builds up, each extra bit of charge must be pushed against a rising voltage. The graph of  $Q$  against  $V$  is a straight line, and the work done is the area under it.



$Q$  against  $V$  for a  $470\ \mu\text{F}$  capacitor. The gradient is  $C$  and the shaded area is the energy stored.

### Key law

Energy stored:  $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$ , with  $E$  in joules,  $Q$  in coulombs,  $C$  in farads and  $V$  in volts.

### Worked example

A  $470\ \mu\text{F}$  capacitor is charged to  $12\ \text{V}$ . Calculate the charge and the stored energy.

$Q = CV = 470 \times 10^{-6} \times 12 = \mathbf{5.64\ mC}$ .  $E = \frac{1}{2}CV^2 = 0.5 \times 470 \times 10^{-6} \times 12^2 = \mathbf{34\ mJ}$ .

## How do capacitors combine in series and in parallel?

| Arrangement | Total capacitance             | What stays equal        |
|-------------|-------------------------------|-------------------------|
| Parallel    | $C = C_1 + C_2 + \dots$       | the voltage across each |
| Series      | $1/C = 1/C_1 + 1/C_2 + \dots$ | the charge on each      |

The rules for capacitors are the reverse of those for resistors: parallel capacitors add, and series capacitors combine through reciprocals. The total in series is always smaller than the smallest single capacitor.

### Worked example

A  $4.7\ \mu\text{F}$  and a  $10\ \mu\text{F}$  capacitor are connected, first in series and then in parallel. Find each total.

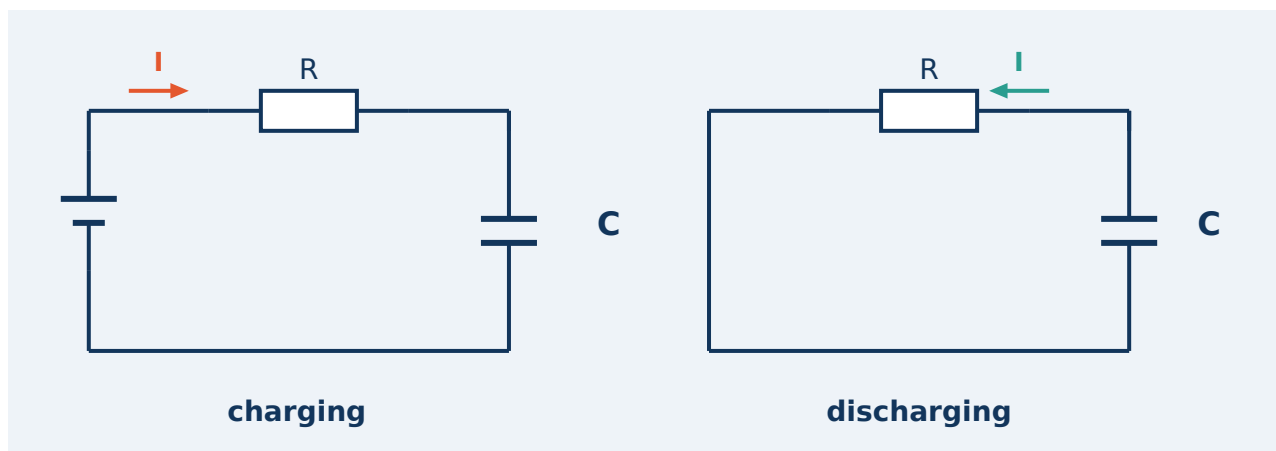
Series:  $1/C = 1/4.7 + 1/10 = 0.3128$ , so  $C = \mathbf{3.2\ \mu F}$ . Parallel:  $C = 4.7 + 10 = \mathbf{14.7\ \mu F}$ .

### Common mistake

Do not copy the resistor rules. If you add series capacitances directly, the total comes out too large, because a series chain behaves like a single capacitor with a wider gap.

## How does a capacitor charge and discharge through a resistor?

A resistor limits the current that flows to or from the capacitor, so the voltage changes gradually. The rate is set by the **time constant**.



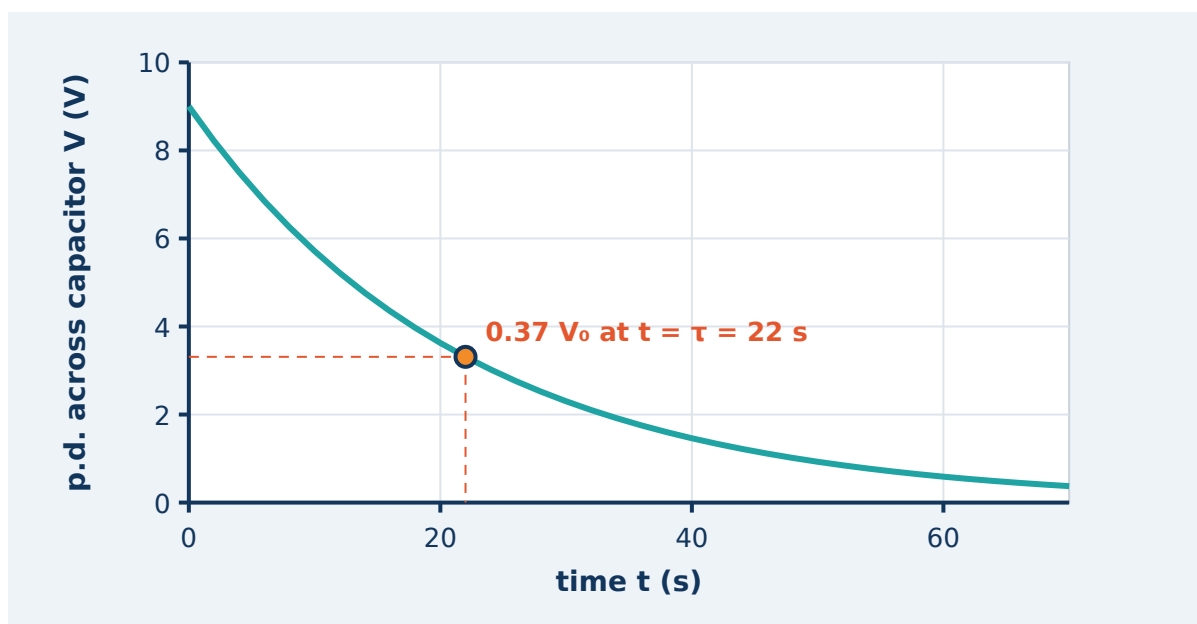
Charging (left): the battery pushes charge through  $R$  onto the plates. Discharging (right): the stored charge flows back through  $R$ .

### Formula

$$\tau = RC$$

$\tau$  = time constant (s),  $R$  = resistance ( $\Omega$ ),  $C$  = capacitance (F). Discharging:  $V = V_0 e^{-t/\tau}$ . Charging:  $V = V_0(1 - e^{-t/\tau})$ , where  $V_0$  is the starting voltage on discharge or the supply voltage on charging.

| Time    | Discharging: $V$ as fraction of $V_0$ | Charging: $V$ as fraction of $V_0$ |
|---------|---------------------------------------|------------------------------------|
| $1\tau$ | 0.37                                  | 0.63                               |
| $2\tau$ | 0.14                                  | 0.86                               |
| $3\tau$ | 0.05                                  | 0.95                               |
| $5\tau$ | 0.007                                 | 0.993                              |



Discharge of a capacitor through a resistor ( $V_0 = 9.0 \text{ V}$ ,  $\tau = 22 \text{ s}$ ). The voltage falls to 37 % after one time constant.

### Worked example

A  $220 \mu\text{F}$  capacitor discharges from  $9.0 \text{ V}$  through  $100 \text{ k}\Omega$ . Find  $V$  after  $30 \text{ s}$  and the time for  $V$  to fall to  $1.0 \text{ V}$ .

$\tau = RC = 100 \times 10^3 \times 220 \times 10^{-6} = 22 \text{ s}$ . After  $30 \text{ s}$ ,  $V = 9.0 e^{-30/22} = \mathbf{2.3 \text{ V}}$ . For  $1.0 \text{ V}$ :  $t = \tau \ln(V_0/V) = 22 \times \ln 9.0 = \mathbf{48 \text{ s}}$ .

### Lina's tip

Count in time constants: a discharging capacitor keeps 37 %, 14 % and 5 % of its voltage after one, two and three time constants. It is practically flat after about  $5\tau$ .

### Key points to remember

- $C = Q / V$ ; 1 farad = 1 coulomb per volt
- Energy stored  $E = \frac{1}{2}CV^2 = \frac{1}{2}QV = Q^2 / 2C$
- Parallel:  $C = C_1 + C_2$ ; series:  $1/C = 1/C_1 + 1/C_2$
- Time constant  $\tau = RC$ ; discharging  $V = V_0 e^{-(t/\tau)}$
- After  $1\tau$  a capacitor has lost 63 % of its charge (discharging) or reached 63 % (charging)

## Charge storage, energy and RC time constants: frequently asked questions

### What does one farad mean?

One farad is one coulomb of charge stored per volt across the capacitor. It is a huge unit, so everyday capacitors have values between picofarads and a few thousand microfarads.

**Why does the current fall as a capacitor charges?**

The voltage across the capacitor rises and opposes the supply voltage, so the voltage across the resistor, and therefore the current, shrinks. The current becomes zero when the capacitor voltage equals the supply voltage.

**What is the time constant of an RC circuit?**

It is  $\tau = RC$ , the time for a discharging capacitor to fall to 37 % of its starting voltage. It is also the time for a charging capacitor to reach 63 % of the supply voltage.

**Why is the energy  $\frac{1}{2}CV^2$  and not  $CV^2$ ?**

The voltage rises from zero as the charge is added, so on average only half of the final voltage acts on each bit of charge. The area under the Q-V graph is a triangle, which gives one half.

**What happens to the energy of a discharging capacitor?**

It is transferred to the resistor as thermal energy as the current flows. For a capacitor discharging completely, the total energy dissipated equals the initial stored energy  $\frac{1}{2}CV^2$ .