

Charge storage, energy and RC time constants

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer gives the formula, the substitution and the result with its unit. Convert microfarads, nanofarads and kilohms before you calculate, and note that final digits may differ slightly if you round at a different step.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.7.4

Problem 1 Charge on a capacitor

$C = 2200 \times 10^{-6} \text{ F}$. $Q = CV = 2200 \times 10^{-6} \times 6.0 = 0.0132 \text{ C}$, so **$Q = 1.3 \times 10^{-2} \text{ C (13 mC)}$** .

Problem 2 Prefix conversions

- $C = Q / V = 2.4 \times 10^{-6} / 12 = \mathbf{2.0 \times 10^{-7} \text{ F}}$.
- $2.0 \times 10^{-7} \text{ F} = \mathbf{0.20 \mu\text{F} = 200 \text{ nF}}$.

Problem 3 Energy in a small capacitor

$E = \frac{1}{2}CV^2 = 0.5 \times 100 \times 10^{-6} \times 25^2 = 0.03125 \text{ J}$, so **$E \approx 3.1 \times 10^{-2} \text{ J (31 mJ)}$** .

Problem 4 Time constant

- $\tau = RC = 47 \times 10^3 \times 100 \times 10^{-6} = \mathbf{4.7 \text{ s}}$.
- $5\tau = 5 \times 4.7 = \mathbf{24 \text{ s}}$ (about 23.5 s).

Problem 5 Two capacitors

- $C = 2.0 + 3.0 = \mathbf{5.0 \mu\text{F}}$.
- $1/C = 1/2.0 + 1/3.0 = 0.8333 \mu\text{F}^{-1}$, so **$C = 1.2 \mu\text{F}$** .

Problem 6 Describing a discharge

- Capacitance is the **charge stored per unit potential difference**, $C = Q / V$.
- Both decay exponentially: they start at their maximum and fall quickly at first, then more and more slowly towards zero.
- After one time constant the voltage is **$e^{-1} \approx 0.37$** of its starting value.

Problem 7 Camera flash

- $Q = CV = 150 \times 10^{-6} \times 300 = \mathbf{0.045 \text{ C}}$.
- $E = \frac{1}{2}CV^2 = 0.5 \times 150 \times 10^{-6} \times 300^2 = \mathbf{6.75 \text{ J}}$.

$$3. P = E / t = 6.75 / 1.0 \times 10^{-3} = \mathbf{6750\ W} \approx \mathbf{6.8 \times 10^3\ W}.$$

Problem 8 Memory back-up

- $\Delta Q = It = 2.0 \times 10^{-3} \times 1.5 = \mathbf{3.0 \times 10^{-3}\ C}.$
- $\Delta V = \Delta Q / C = 3.0 \times 10^{-3} / 1000 \times 10^{-6} = \mathbf{3.0\ V}.$
- $V = 9.0 - 3.0 = \mathbf{6.0\ V}.$

Problem 9 Three in series

- $1/C = 1/6.0 + 1/3.0 + 1/2.0 = (1 + 2 + 3)/6.0 = 1.0\ \mu\text{F}^{-1}$, so $\mathbf{C = 1.0\ \mu\text{F}}.$
- The same charge is on each: $Q = CV = 1.0 \times 10^{-6} \times 12 = \mathbf{12\ \mu\text{C}}.$
- $V = Q / C$ gives $\mathbf{2.0\ V}$ (6.0 μF), $\mathbf{4.0\ V}$ (3.0 μF) and $\mathbf{6.0\ V}$ (2.0 μF), which add to 12 V.

Problem 10 Network of capacitors

- $1/C = 1/6.0 + 1/3.0 = 0.5\ \mu\text{F}^{-1}$, so $\mathbf{C = 2.0\ \mu\text{F}}.$
- The branches are in parallel: $C = 4.0 + 2.0 = \mathbf{6.0\ \mu\text{F}}.$
- $Q = CV = 6.0 \times 10^{-6} \times 9.0 = \mathbf{54\ \mu\text{C}}.$ $E = \frac{1}{2}CV^2 = 0.5 \times 6.0 \times 10^{-6} \times 9.0^2 = \mathbf{243\ \mu\text{J}}$ ($2.4 \times 10^{-4}\ \text{J}$).

Problem 11 Discharge calculation

- $\tau = RC = 22 \times 10^3 \times 470 \times 10^{-6} = \mathbf{10.3\ s}.$
- $V = V_0 e^{-t/\tau} = 12 e^{-15/10.34} = 12 \times 0.2344 = \mathbf{2.8\ V}.$
- $I_0 = V_0 / R = 12 / 22 \times 10^3 = \mathbf{0.55\ mA}.$

Problem 12 Finding τ from a graph

- $V_0 = 8.0\ \text{V}$ and $0.37 \times 8.0 = \mathbf{2.96\ V} \approx \mathbf{3.0\ V}$ ($e^{-1} \times 8.0 = 2.94\ \text{V}$).
- The graph passes through 2.94 V at $t = 10\ \text{s}$, so $\mathbf{\tau = 10\ s}.$
- $C = \tau / R = 10 / (10 \times 10^3) = \mathbf{1.0 \times 10^{-3}\ F}$ ($\mathbf{1000\ \mu\text{F}}$).

Problem 13 Charging a capacitor

- $\tau = 4.7 \times 10^3 \times 2200 \times 10^{-6} = \mathbf{10.34\ s}.$
- $V = V_0(1 - e^{-t/\tau}) = 9.0 (1 - e^{-10/10.34}) = 9.0 \times 0.6198 = \mathbf{5.6\ V}.$
- $6.0 = 9.0 (1 - e^{-t/\tau})$ gives $e^{-t/\tau} = 1/3$, so $t = \tau \ln 3 = 10.34 \times 1.0986 = \mathbf{11.4\ s}.$

Problem 14 Spot the error

- The values were not converted to ohms and farads ($2.2\ \text{k}\Omega = 2.2 \times 10^3\ \Omega$ and $470\ \mu\text{F} = 470 \times 10^{-6}\ \text{F}$), and after one time constant the capacitor is still at 37 % of its voltage, not zero.
- $\tau = 2.2 \times 10^3 \times 470 \times 10^{-6} = \mathbf{1.03\ s}.$

3. $V = 0.37 \times 6.0 = \mathbf{2.2\ V}$ (exactly $6.0\ \text{e}^{-1} = 2.21\ \text{V}$).

Problem 15 Parallel-plate capacitor

1. $C = \epsilon_0 A / d = (8.85 \times 10^{-12} \times 0.050) / (0.50 \times 10^{-3}) = \mathbf{8.8 \times 10^{-10}\ F\ (0.89\ nF)}$.

2. $Q = CV = 8.85 \times 10^{-10} \times 100 = \mathbf{8.8 \times 10^{-8}\ C}$ (89 nC).

3. C is inversely proportional to d, so halving d **doubles the capacitance**.

Problem 16 Sharing charge

1. $Q = CV = 4.0 \times 10^{-6} \times 12 = \mathbf{48\ \mu C}$.

2. Charge is conserved and the capacitors are now in parallel: $V = Q / (C_1 + C_2) = 48 / 6.0 = \mathbf{8.0\ V}$.

3. $E_{\text{before}} = \frac{1}{2} \times 4.0 \times 10^{-6} \times 12^2 = 288\ \mu\text{J}$ and $E_{\text{after}} = \frac{1}{2} \times 6.0 \times 10^{-6} \times 8^2 = 192\ \mu\text{J}$. Loss = **96 μJ** , transferred as thermal energy in the connecting wires and by radiation as charge moves.

Problem 17 Energy and time constant

1. $E_0 = \frac{1}{2} CV_0^2 = 0.5 \times 1000 \times 10^{-6} \times 10^2 = \mathbf{0.05\ J}$.

2. $E = \frac{1}{2} CV^2$ and $V = V_0 e^{-t/\tau}$, so $E = \frac{1}{2} CV_0^2 e^{-2t/\tau} = E_0 e^{-2t/\tau}$.

3. $\tau = RC = 1.0\ \text{s}$. $0.25 = e^{-2t/\tau}$ gives $2t/\tau = \ln 4$, so $t = \tau \ln 2 = \mathbf{0.69\ s}$. Then $V = 10\ \text{e}^{-0.693} = \mathbf{5.0\ V}$, half the starting voltage, as expected for a quarter of the energy.

Problem 18 Delay timer design

1. $V = V_0(1 - e^{-t/RC}) = 6.0$ with $V_0 = 9.0$ gives $e^{-t/RC} = 1/3$, so $t = RC \ln 3$.

2. $R = t / (C \ln 3) = 30 / (470 \times 10^{-6} \times 1.0986) = \mathbf{58\ k\Omega}$ ($5.8 \times 10^4\ \Omega$).

3. $t = 56 \times 10^3 \times 470 \times 10^{-6} \times 1.0986 = \mathbf{28.9\ s}$, about 29 s.

Problem 19 Linearising the decay

1. $\ln V = 2.303, 1.803, 1.303, 0.803, 0.303$.

2. $V = V_0 e^{-t/\tau}$ gives $\ln V = \ln V_0 - t/\tau$, which has the form $y = c + mx$ with gradient $-1/\tau$.
Gradient = $(0.303 - 2.303) / 40 = \mathbf{-0.050\ s^{-1}}$.

3. $\tau = 1 / 0.050 = \mathbf{20\ s}$. $C = \tau / R = 20 / (40 \times 10^3) = \mathbf{5.0 \times 10^{-4}\ F\ (500\ \mu F)}$.

Problem 20 Defibrillator pulse

1. $E = \frac{1}{2} CV^2 = 0.5 \times 32 \times 10^{-6} \times (5.0 \times 10^3)^2 = \mathbf{400\ J}$.

2. $I_0 = V / R = 5000 / 50 = \mathbf{100\ A}$. $\tau = RC = 50 \times 32 \times 10^{-6} = \mathbf{1.6\ ms}$.

3. The energy remaining is $E_0 e^{-2t/\tau}$, so the fraction delivered is $1 - e^{-2 \times 2.0/1.6} = 1 - e^{-2.5} = 0.9179$. Energy = $400 \times 0.9179 = \mathbf{367\ J \approx 3.7 \times 10^2\ J}$.