

Storing charge and energy between two plates

AP Physics 2 (algebra-based) · Lesson

Quick answer: A capacitor stores charge on two plates separated by an insulator, with capacitance $C = Q/V$ in farads. For parallel plates $C = \kappa\epsilon_0 A/d$. The stored energy is $U = \frac{1}{2}CV^2$, and a dielectric of constant κ multiplies the capacitance by κ by reducing the field between the plates.

Level: AP Physics 2 (algebra-based) · **Curriculum:** AP P2 Unit 10

What is a capacitor?

A capacitor is two conductors separated by an insulator. When you connect it to a battery, electrons flow onto one plate and leave the other, so the plates hold equal and opposite charges $+Q$ and $-Q$. The voltage across the plates is set by the battery. Capacitors sit inside phone touchscreens, camera flashes and radio tuners, where they store charge and energy briefly and release it quickly.

Definition

Capacitance is the charge stored per volt, $C = Q/V$, measured in farads ($1 \text{ F} = 1 \text{ C/V}$). Practical capacitors are microfarads (10^{-6} F), nanofarads (10^{-9} F) or picofarads (10^{-12} F). Q means the size of the charge on one plate.

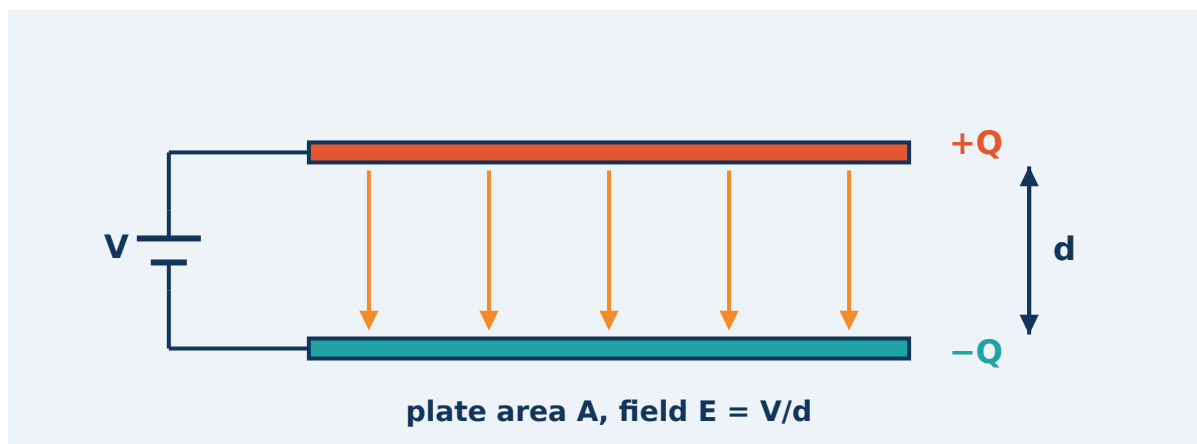
How does a parallel-plate capacitor depend on its shape?

Large plates hold more charge, so capacitance grows with area A . Plates that are close together make a strong field for the same voltage and so can hold more charge, so capacitance falls as the gap d grows. This leads to a simple geometric rule.

Formula

$$C = \kappa\epsilon_0 A / d$$

C is the capacitance (F), κ the dielectric constant (no unit; $\kappa = 1$ for vacuum), $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ the permittivity of free space, A the area of one plate (m^2) and d the plate separation (m).



A parallel-plate capacitor: plates of area A separated by a gap d and connected to a battery of voltage V .

Worked example

Wanjiru builds a capacitor from two square plates of side 10 cm, 1.0 mm apart in air, and connects it to a 12 V battery. Find C , Q and the stored energy.

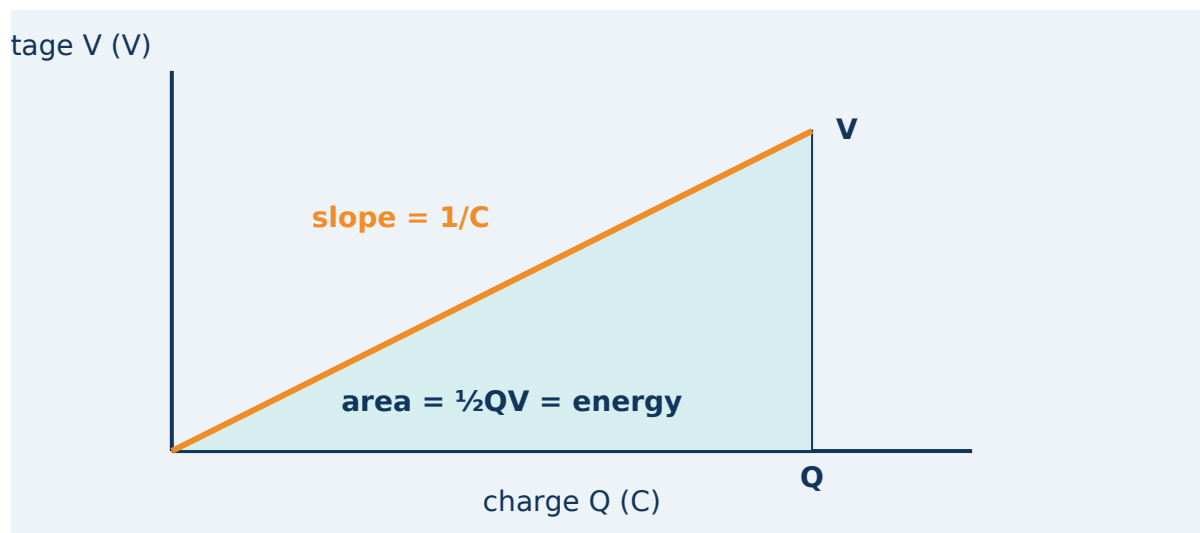
$A = 0.10 \times 0.10 = 1.0 \times 10^{-2} \text{ m}^2$. $C = (8.85 \times 10^{-12})(1.0 \times 10^{-2}) / (1.0 \times 10^{-3}) = \mathbf{8.9 \times 10^{-11} \text{ F}}$ (89 pF). $Q = CV = 8.85 \times 10^{-11} \times 12 = \mathbf{1.1 \times 10^{-9} \text{ C}}$. $U = \frac{1}{2}CV^2 = 0.5 \times 8.85 \times 10^{-11} \times 144 = \mathbf{6.4 \times 10^{-9} \text{ J}}$.

How much energy does a capacitor store?

To charge a capacitor, work is done to push each extra bit of charge against the voltage already on the plates. The voltage rises in proportion to the charge, so the graph of V against Q is a straight line. The energy equals the area under it, which is a triangle.

Key law

$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2 / 2C$. The energy is stored in the electric field between the plates. Doubling the voltage on a fixed capacitor makes the energy four times larger.



The area of the shaded triangle is the energy stored in the capacitor.

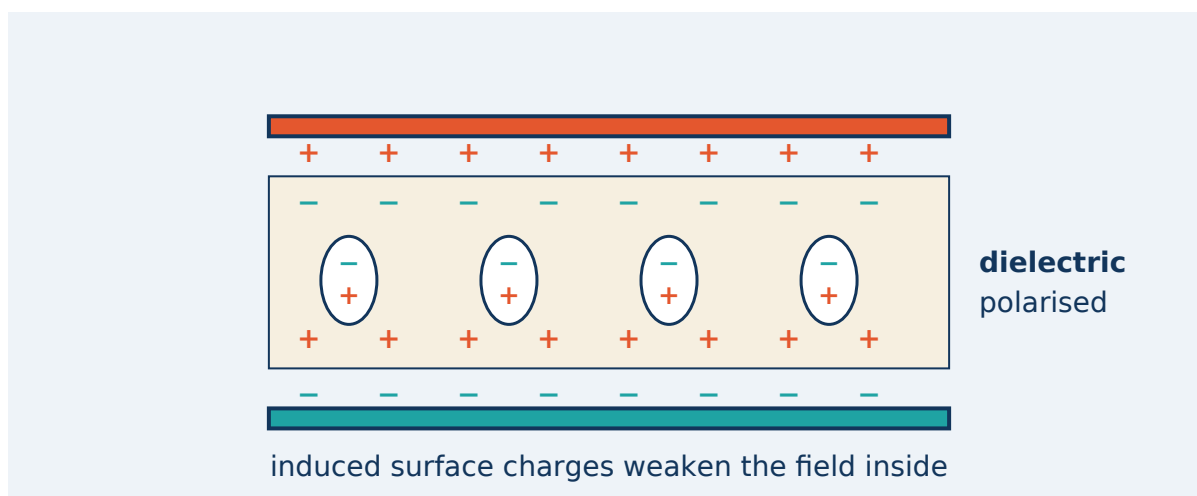
Worked example

The flash in Aarav's camera uses a $330 \mu\text{F}$ capacitor charged to 300 V . Find the energy and the average power if it discharges in 2.0 ms .

$U = \frac{1}{2}CV^2 = 0.5 \times 330 \times 10^{-6} \times 300^2 = \mathbf{15 \text{ J}}$. $P = U/t = 14.85 / 2.0 \times 10^{-3} = \mathbf{7.4 \times 10^3 \text{ W}}$.
A small battery cannot give that power directly, but the capacitor can.

What does a dielectric do?

A dielectric is an insulator placed between the plates. Its molecules turn in the field, and induced charge forms on its surfaces that opposes the field of the plates. The net field is smaller, so for the same charge the voltage drops, and the capacitance rises by the factor κ .



In a dielectric the molecules line up with the field, leaving induced charges that partly cancel the field.

Material	Dielectric constant κ
Vacuum	1 (exactly)
Air	1.0006
Polythene	2.3
Paper	3.7
Glass	6
Pure water (20 °C)	80

What happens next depends on whether the battery stays connected. With the battery connected, V is fixed, so C rises and so do Q and U . If the capacitor is isolated, Q is fixed, so C rises but V and U fall.

Worked example

The 89 pF capacitor above has a sheet of paper ($\kappa = 3.7$) slid between the plates while the 12 V battery stays connected. Find the new C , Q and U .

$C = 3.7 \times 88.5 = \mathbf{3.3 \times 10^2 \text{ pF}}$. $Q = 3.27 \times 10^{-10} \times 12 = \mathbf{3.9 \times 10^{-9} \text{ C}}$. $U = \frac{1}{2}CV^2 = 0.5 \times 3.27 \times 10^{-10} \times 144 = \mathbf{2.4 \times 10^{-8} \text{ J}}$, which is 3.7 times the earlier energy.

Common mistake

Do not say that a capacitor stores a net charge. The plates carry $+Q$ and $-Q$, so the total is zero. Also check which quantity is fixed before inserting a dielectric: the battery fixes V , while disconnection fixes Q .

Lina's tip

Before changing a capacitor, write down what stays fixed: V if the battery is still connected, Q if it is not. Then use $C = Q/V$ to find the other quantity.

Key points to remember

- Capacitance $C = Q/V$ in farads; one plate holds $+Q$ and the other $-Q$
- Parallel plates: $C = \kappa \epsilon_0 A/d$, with $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
- Energy stored: $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/2C$
- A dielectric multiplies C by κ ; battery connected fixes V , disconnected fixes Q
- Too strong a field makes the dielectric break down and spark

Storing charge and energy between two plates: frequently asked questions

What is a farad?

A farad is one coulomb of stored charge per volt across the capacitor. It is a very large unit, so most capacitors are given in microfarads, nanofarads or picofarads.

Why does a dielectric increase capacitance?

Its molecules polarise and create a field opposite to the field of the plates. The net field drops, so for the same charge the voltage is smaller, which means a larger Q/V .

Where is the energy stored in a capacitor?

In the electric field in the space between the plates. A stronger field and a larger volume store more energy, and the energy is $U = \frac{1}{2}CV^2$.

What happens if the plates of a charged, isolated capacitor are pulled apart?

The charge stays the same but C falls, so V rises. Energy increases because you do work pulling the opposite charges apart.

Why do capacitors have a maximum voltage rating?

If the field becomes too strong, the dielectric ionises and conducts, causing a spark or permanent damage. The rating keeps the field below the breakdown value.