

# Storing charge and energy between two plates

AP Physics 2 (algebra-based) · Worksheet answers

Check your method as well as your numbers. In AP-style marking, you earn credit for stating what is held constant, writing the right relation, substituting with units and giving a conclusion. Rounding differences in the third significant figure are acceptable.

**Level:** AP Physics 2 (algebra-based) · **Curriculum:** AP P2 Unit 10

## Problem 1 Capacitance from charge

1.  $C = Q/V = 24 / 6.0 = \mathbf{4.0 \mu F}$ .
2.  $Q = CV = 4.0 \times 9.0 = \mathbf{36 \mu C}$ .
3. One farad is one coulomb per volt:  $1 \text{ F} = 1 \text{ C/V}$ .

## Problem 2 Prefixes

1.  $\mathbf{4.7 \times 10^{-4} \text{ F}}$ .
2.  $\mathbf{2.2 \times 10^{-8} \text{ F}}$ .
3.  $100 \times 10^{-12} \text{ F} = \mathbf{1.0 \times 10^{-4} \mu F}$ .
4.  $\mathbf{3.3 \text{ mF}}$ .

## Problem 3 Plate capacitor in air

1.  $C = \epsilon_0 A/d = (8.85 \times 10^{-12})(0.020) / (5.0 \times 10^{-4}) = 3.54 \times 10^{-10} \text{ F} = \mathbf{354 \text{ pF}}$ .
2. C is inversely proportional to d, so it **halves** to 177 pF.

## Problem 4 Energy in a capacitor

1.  $Q = CV = 100 \times 10^{-6} \times 12 = \mathbf{1.2 \times 10^{-3} \text{ C}}$ .
2.  $U = \frac{1}{2}CV^2 = 0.5 \times 100 \times 10^{-6} \times 144 = \mathbf{7.2 \times 10^{-3} \text{ J}}$ .

## Problem 5 Electrons on the plates

1.  $Q = CV = 2.2 \times 10^{-6} \times 9.0 = \mathbf{2.0 \times 10^{-5} \text{ C}}$  (19.8  $\mu\text{C}$ ), negative.
2.  $N = 1.98 \times 10^{-5} / 1.60 \times 10^{-19} = \mathbf{1.2 \times 10^{14}}$  electrons.

## Problem 6 Scaling capacitance

1.  $\mathbf{2C}$ .
2.  $\mathbf{C/2}$ .
3.  $\mathbf{3C}$ .
4. Factors  $2 \times (1/2) \times 3 = 3$ , so  $\mathbf{3C}$ .

**Problem 7 Finding the gap**

1.  $d = \epsilon_0 A / C$ .
2.  $d = (8.85 \times 10^{-12})(0.050) / 1.0 \times 10^{-9} = 4.4 \times 10^{-4} \text{ m} = \mathbf{0.44 \text{ mm}}$ .

**Problem 8 Charging graph**

1. Gradient =  $10 / 40 \mu\text{C} = 0.25 \text{ V}/\mu\text{C} = 2.5 \times 10^5 \text{ V/C}$ , which is  $1/C$ , so  $C = \mathbf{4.0 \mu\text{F}}$ .
2.  $U = \frac{1}{2}QV = 0.5 \times 40 \times 10^{-6} \times 10 = \mathbf{2.0 \times 10^{-4} \text{ J}}$ .
3. The area under the graph is the energy stored, the work done in charging.

**Problem 9 Three energy formulas**

1.  $V = Q/C = 50 / 10 = \mathbf{5.0 \text{ V}}$ .
2.  $U = (50 \times 10^{-6})^2 / (2 \times 10 \times 10^{-6}) = \mathbf{1.25 \times 10^{-4} \text{ J}}$ .
3.  $\frac{1}{2}CV^2 = 0.5 \times 10 \times 10^{-6} \times 25 = 1.25 \times 10^{-4} \text{ J}$ , which matches.

**Problem 10 Dielectric with the battery connected**

1.  $C = 4.0 \times 2.0 = \mathbf{8.0 \text{ nF}}$ .
2. Before:  $2.0 \times 10^{-9} \times 9.0 = 18 \text{ nC}$ . After:  $8.0 \times 10^{-9} \times 9.0 = \mathbf{72 \text{ nC}}$ .
3. Before:  $\frac{1}{2}CV^2 = 8.1 \times 10^{-8} \text{ J}$ . After:  $\mathbf{3.2 \times 10^{-7} \text{ J}}$ .
4.  $V$  and  $d$  are fixed, so  $E = V/d$  is **unchanged** overall.

**Problem 11 Dielectric in an isolated capacitor**

1. Isolated, so  $Q$  stays  $\mathbf{18 \text{ nC}}$ .
2.  $V = Q/C = 18 / 8.0 = \mathbf{2.25 \text{ V}}$ .
3.  $U = Q^2/2C = (18 \times 10^{-9})^2 / (16 \times 10^{-9}) = \mathbf{2.0 \times 10^{-8} \text{ J}}$ , a quarter of the original.
4. The capacitor does work pulling the dielectric in, so energy goes into kinetic energy of the slab and, if it is stopped, thermal energy.

**Problem 12 Supercapacitor torch**

1.  $U = \frac{1}{2}CV^2 = 0.5 \times 0.50 \times 25 = \mathbf{6.3 \text{ J}}$  (6.25 J).
2.  $t = U/P = 6.25 / 0.25 = \mathbf{25 \text{ s}}$ .
3. At 3.0 V,  $U = 0.5 \times 0.50 \times 9 = 2.25 \text{ J}$ . Released:  $6.25 - 2.25 = \mathbf{4.0 \text{ J}}$ .

**Problem 13 Air breakdown**

1.  $V = Ed = 3.0 \times 10^6 \times 2.0 \times 10^{-3} = \mathbf{6.0 \times 10^3 \text{ V}}$ .
2.  $C = (8.85 \times 10^{-12})(0.040) / (2.0 \times 10^{-3}) = \mathbf{1.8 \times 10^{-10} \text{ F}}$  (177 pF).
3.  $Q = CV = 1.77 \times 10^{-10} \times 6000 = \mathbf{1.1 \times 10^{-6} \text{ C}}$ ;  $U = \frac{1}{2}QV = \mathbf{3.2 \times 10^{-3} \text{ J}}$ .

### Problem 14 Inside a dielectric

1. They polarise: the molecules turn so that their positive ends point towards the negative plate and the negative ends towards the positive plate.
2. Induced charges on the glass surfaces create a field opposite to the field of the plates, so the net field is smaller.
3. A smaller field means a smaller voltage for the same  $Q$ , and  $C = Q/V$  is **larger by the factor  $\kappa$** .

### Problem 15 Measuring a dielectric constant

1.  $\kappa = C_{\text{filled}} / C_{\text{air}} = 190 / 52 = \mathbf{3.7}$ .
2. It resembles **paper** ( $\kappa = 3.7$ ).
3. An air gap between the sheet and the plates, or stray capacitance in the leads, changes the reading.

### Problem 16 Pulling the plates apart

1.  $C$  halves to  **$2.5 \mu\text{F}$**  in both.
2. A:  $V = 12 \text{ V}$ ,  $Q = 2.5 \times 12 = 30 \mu\text{C}$ ,  $U = \frac{1}{2}CV^2 = \mathbf{1.8 \times 10^{-4} \text{ J}}$  (down from  $3.6 \times 10^{-4} \text{ J}$ ).
3. B:  $Q = 60 \mu\text{C}$ ,  $V = Q/C = 24 \text{ V}$ ,  $U = \frac{1}{2}QV = \mathbf{7.2 \times 10^{-4} \text{ J}}$  (up).
4. In B you do work pulling opposite charges apart, and the charge cannot leave, so  $U$  doubles. In A the battery takes charge back as  $C$  falls, so  $U$  falls.

### Problem 17 Partly filled capacitors

1.  $C_0 = (8.85 \times 10^{-12})(0.020) / 1.0 \times 10^{-3} = \mathbf{177 \text{ pF}}$ .
2. Two capacitors in parallel, each with half the area:  $C = \frac{1}{2}C_0 + \frac{1}{2}(5C_0) = 3C_0 = \mathbf{531 \text{ pF}}$ .
3. Two capacitors in series, each with half the gap:  $1/C = 1/(2C_0) + 1/(10C_0) = 6/(10C_0)$ , so  $C = (5/3)C_0 = \mathbf{295 \text{ pF}}$ .
4. Both are larger than  $C_0$ , but the side-by-side slab raises  $C$  more than the stacked slab with the same volume of dielectric.

### Problem 18 Sharing charge

1.  $Q = CV = 40 \mu\text{C}$ ;  $U = \frac{1}{2}CV^2 = 0.5 \times 4.0 \times 10^{-6} \times 100 = \mathbf{2.0 \times 10^{-4} \text{ J}}$ .
2. The charge is conserved and the capacitors are in parallel:  $V = 40 / (4.0 + 6.0) = \mathbf{4.0 \text{ V}}$ .
3.  $U = \frac{1}{2}(10 \times 10^{-6})(16) = 8.0 \times 10^{-5} \text{ J}$ . Lost:  $\mathbf{1.2 \times 10^{-4} \text{ J}}$  (60 %).
4. Heating of the connecting wires and switch, a tiny spark, and radiated electromagnetic waves.

### Problem 19 Defibrillator capacitor

1.  $U = \frac{1}{2}CV^2 = 0.5 \times 80 \times 10^{-6} \times (3000)^2 = \mathbf{3.6 \times 10^2 \text{ J}}$ .
2. Delivered:  $0.80 \times 360 = 288 \text{ J}$ ;  $P = 288 / 5.0 \times 10^{-3} = \mathbf{5.8 \times 10^4 \text{ W}}$ .

3.  $Q = CV = 80 \times 10^{-6} \times 3000 = 0.24 \text{ C}$ ;  $I = Q/t = 0.24 / 5.0 \times 10^{-3} = \mathbf{48 \text{ A}}$ .

### Problem 20 Doubling the voltage

1.  $Q = CV$ , so the second capacitor holds **twice the charge**.
2.  $U = \frac{1}{2}CV^2$ , so the second stores **four times the energy**.
3.  $E = V/d$  with the same  $d$ , so the field is **twice as large**.
4. The second one, because its field is twice as close to the breakdown value of the dielectric.