

Centre of mass, impulse and momentum with calculus

AP Physics C (Mechanics and E&M, calculus-based) · Test answers (mark scheme)

Model answers with marking guidance. Method marks are given for a correct integral or conservation equation even when a later arithmetic slip occurs. Always require units on final answers, and apply error carried forward where a later part uses an earlier wrong value.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 4

Question 1 Impulse and momentum (2 marks)

a. Impulse is the integral of force over time, $J = \int F \, dt$. b. It **equals the change in momentum**, $J = \Delta p$.

Marking: 1 mark each.

Question 2 Centre of mass of a rod (3 marks)

a. $M = \int_0^{0.80} 5.0x^2 \, dx = (5/3)(0.512) = \mathbf{0.853 \, kg}$. b. $\int x\lambda \, dx = \int 5.0x^3 \, dx = (5/4)(0.4096) = 0.512$. c. $x_{cm} = 0.512/0.853 = \mathbf{0.60 \, m}$.

Marking: 1 mark for each part.

Question 3 Force-time graph (3 marks)

a. Area = triangle + rectangle + triangle = $\frac{1}{2} \times 0.002 \times 600 + 0.004 \times 600 + \frac{1}{2} \times 0.002 \times 600 = 0.6 + 2.4 + 0.6 = \mathbf{3.6 \, N \cdot s}$. b. $v = J/m = 3.6/0.057 = \mathbf{63 \, m/s}$.

Marking: 1 mark for the shape split, 1 for the area; 1 mark for speed.

Question 4 Sticking collision (3 marks)

a. $0.80 \times 1.5 = 1.2v$, so $v = \mathbf{1.0 \, m/s}$. b. $K_i = \frac{1}{2} \times 0.80 \times 2.25 = 0.90 \, J$; $K_f = \frac{1}{2} \times 1.2 \times 1.0 = 0.60 \, J$. c. Loss = $\mathbf{0.30 \, J}$.

Marking: 1 mark per part.

Question 5 Elastic collision (3 marks)

a. Momentum and kinetic energy are conserved. b. $v_1 = (2.0 - 1.0)/(3.0) \times 3.0 = \mathbf{1.0 \, m/s}$; $v_2 = (2 \times 2.0)/(3.0) \times 3.0 = \mathbf{4.0 \, m/s}$.

Marking: 1 mark for the two laws; 1 mark for each speed.

Question 6 Rocket (2 marks)

a. $\Delta v = v_e \ln(m_0/m)$. b. $\Delta v = 2500 \times \ln 2 = 2500 \times 0.693 = \mathbf{1.7 \times 10^3 \text{ m/s}}$.

Marking: 1 mark each.

Question 7 Exploding firework (2 marks)

The explosion forces are internal, and the only external force is gravity. The centre of mass therefore keeps following the same **parabola** as the unexploded firework would have, until pieces land.

Marking: 1 mark for internal forces; 1 mark for the unchanged path.

Question 8 Padded gloves (2 marks)

The change of momentum of the ball is fixed, so the impulse is fixed. Padding and moving the hand back increase the contact time, so the average force $J/\Delta t$ on the hand is smaller.

Longer time, smaller force.

Marking: 1 mark for fixed impulse; 1 mark for longer time giving smaller force.