

Centre of mass, impulse and momentum with calculus

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These problems cover centre of mass for discrete and continuous bodies, impulse as an area or an integral, and conservation of momentum in collisions, explosions and variable-mass motion. Early items check definitions and units; later items need integration and vectors. They suit AP Physics C: Mechanics unit 4.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 4

Problem 1 Momentum of a cricket ball (easy)

A cricket ball of mass 0.16 kg travels at 35 m/s towards the batter.

1. Calculate its momentum.
2. State its direction.

Problem 2 Impulse of a constant force (easy)

A constant force of 50 N acts for 0.20 s on a 2.0 kg trolley.

1. Calculate the impulse.
2. Calculate the change in speed.

Problem 3 Two-mass centre (easy)

A 2.0 kg mass sits at $x = 0$ and a 6.0 kg mass at $x = 4.0$ m.

1. Calculate the centre of mass.

Problem 4 Units of impulse (easy)

1. Show that the newton-second and $\text{kg}\cdot\text{m/s}$ are the same unit.
2. State which is more natural for impulse and which for momentum.

Problem 5 Skaters pushing apart (easy)

Wanjiru (50 kg) and Aarav (70 kg) stand at rest on smooth ice and push apart. Wanjiru moves away at 2.8 m/s.

1. Calculate Aarav's speed.
2. State his direction.

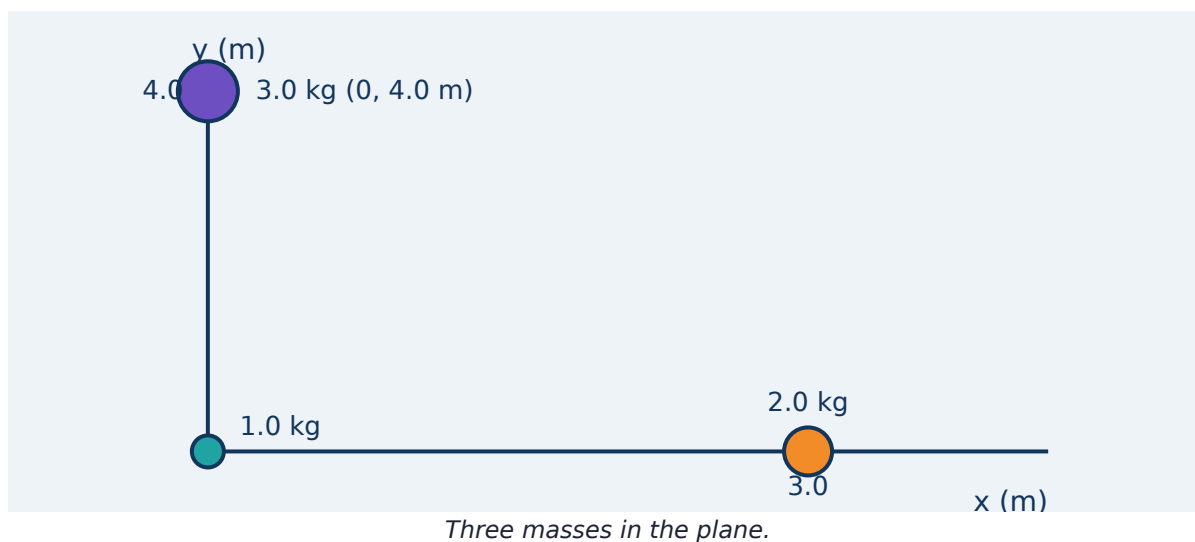
Problem 6 Egg and pillow (easy)

An egg falling at 5.0 m/s stops on a pillow instead of a stone floor.

1. Explain, using impulse, why the egg is less likely to break on the pillow.

Problem 7 Three-mass centre (medium)

Masses of 1.0 kg at (0, 0), 2.0 kg at (3.0 m, 0) and 3.0 kg at (0, 4.0 m).



1. Calculate x_{cm} .
2. Calculate y_{cm} .

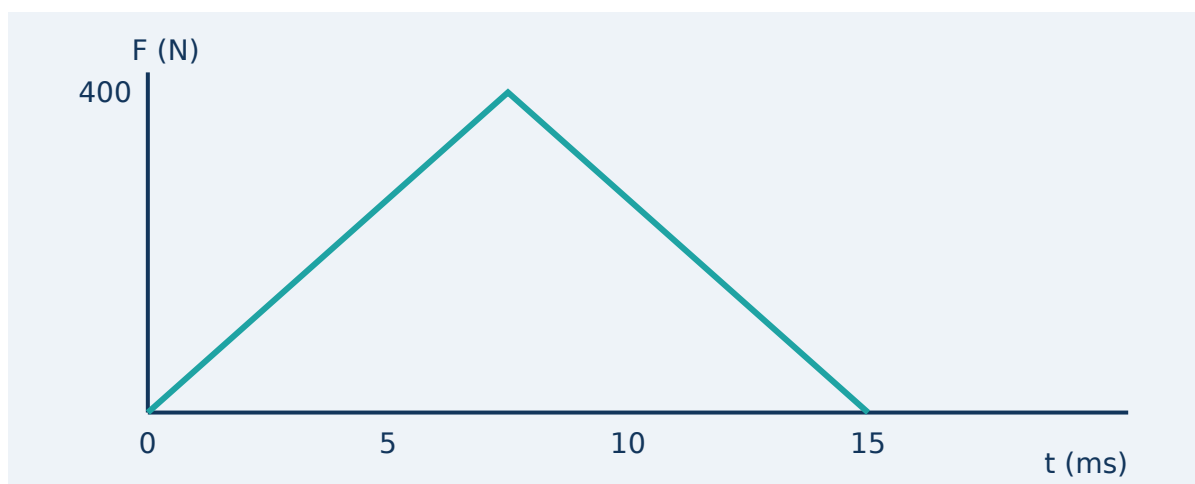
Problem 8 Non-uniform rod (medium)

A rod of length 2.0 m has $\lambda = 3.0x$ kg/m, with x measured from one end.

1. Calculate its mass.
2. Calculate x_{cm} by integration.

Problem 9 Impulse from a graph (medium)

The graph shows the force on a 0.060 kg tennis ball during a serve. The ball starts at rest.



Force on the ball during a serve.

1. Calculate the impulse from the area.
2. Calculate the launch speed.

Problem 10 Cars locking together (medium)

A 1200 kg car at 15 m/s hits a stationary 800 kg car and they lock together.

1. Calculate the common speed.
2. Calculate the kinetic energy lost.

Problem 11 Elastic collision on a track (medium)

A 0.30 kg trolley at 4.0 m/s hits a 0.10 kg trolley at rest in an elastic collision.

1. Calculate the final velocity of each trolley.

Problem 12 Exploding shell (medium)

A 4.0 kg shell moving horizontally at 20 m/s at the top of its path splits into a 1.0 kg piece, which stops, and a 3.0 kg piece.



Shell before and after the explosion.

1. Calculate the speed of the 3.0 kg piece.
2. State where the centre of mass of the pieces lands.

Problem 13 Impulse mistake (medium)

Bilal in Lahore says: "A bat pushes the ball with 800 N while the bat moves 0.50 m, so the impulse is $800 \times 0.50 = 400 \text{ N}\cdot\text{s}$."

1. Identify the error.
2. Find the impulse if the contact lasts 4.0 ms with this constant force.

Problem 14 Time-varying force (medium)

A force $F = 6.0t^2$ newtons acts on a 4.0 kg mass at rest from $t = 0$ to $t = 2.0 \text{ s}$.

1. Calculate the impulse by integration.
2. Calculate the final speed.

Problem 15 Semicircular plate (hard)

A thin uniform plate is a half disc of radius $R = 0.30 \text{ m}$ and surface density σ .

1. Take a horizontal strip at height y above the diameter and write dm .
2. Integrate to show $y_{\text{cm}} = 4R/(3\pi)$.
3. Calculate y_{cm} .

Problem 16 Density rising linearly (hard)

A rod of length 0.90 m has $\lambda = \lambda_0(1 + x/L)$.

1. Show that $x_{\text{cm}} = 5L/9$.
2. Calculate x_{cm} .

Problem 17 Rocket burn (hard)

Kofi in Accra models a rocket of initial mass 500 kg, which burns 350 kg of fuel with exhaust speed 2000 m/s relative to the rocket. Ignore gravity and drag.

1. Calculate the final speed gained.
2. The fuel burns at 5.0 kg/s. Calculate the thrust.

Problem 18 Sand on a belt (hard)

Sand falls vertically onto a horizontal belt moving at a constant 0.50 m/s at a rate of 8.0 kg/s.

1. Calculate the extra horizontal force the motor must supply.
2. Calculate the power it supplies.
3. Calculate the rate of increase of the sand's kinetic energy and explain the difference.

Problem 19 Ballistic pendulum (hard)

A 10 g bullet embeds in a 2.0 kg block hanging on strings, which swings up 0.12 m.

1. Calculate the speed of block and bullet just after impact.

2. Calculate the bullet's speed before impact.

Problem 20 Billiard balls at an angle (hard)

A 0.17 kg ball at 4.0 m/s strikes an identical stationary ball. Afterwards the first moves at 3.2 m/s at 37° above its original direction (take $\cos 37^\circ = 0.80$, $\sin 37^\circ = 0.60$).

1. Find the velocity components of the second ball.
2. Calculate its speed and direction.
3. Decide whether the collision was elastic.