

Centre of mass, impulse and momentum with calculus

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Every answer states the principle first, then the substitution and the result with units. For the integration problems each step is shown so you can compare your own working. Take $g = 9.81 \text{ m/s}^2$ and give results to two or three significant figures.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 4

Problem 1 Momentum of a cricket ball

1. $p = mv = 0.16 \times 35 = \mathbf{5.6 \text{ kg}\cdot\text{m/s}}$.
2. The momentum has the **same direction as the velocity**, towards the batter.

Problem 2 Impulse of a constant force

1. $J = F\Delta t = 50 \times 0.20 = \mathbf{10 \text{ N}\cdot\text{s}}$.
2. $\Delta v = J/m = 10/2.0 = \mathbf{5.0 \text{ m/s}}$.

Problem 3 Two-mass centre

$x_{\text{cm}} = (2.0 \times 0 + 6.0 \times 4.0)/(2.0 + 6.0) = 24/8.0 = \mathbf{3.0 \text{ m}}$, closer to the heavier mass.

Problem 4 Units of impulse

1. $\text{N}\cdot\text{s} = (\text{kg}\cdot\text{m/s}^2) \times \text{s} = \mathbf{\text{kg}\cdot\text{m/s}}$.
2. $\text{N}\cdot\text{s}$ is natural when force and time are given (impulse); $\text{kg}\cdot\text{m/s}$ when mass and velocity are given (momentum).

Problem 5 Skaters pushing apart

1. Total momentum is zero: $50 \times 2.8 - 70v = 0$, so $v = 140/70 = \mathbf{2.0 \text{ m/s}}$.
2. He moves in the **opposite direction** to Wanjiru.

Problem 6 Egg and pillow

The change of momentum is the same, so the impulse is the same. The pillow lengthens the stopping time, and since $F = J/\Delta t$ the **average force is smaller**.

Problem 7 Three-mass centre

1. $x_{\text{cm}} = (0 + 6.0 + 0)/6.0 = \mathbf{1.0 \text{ m}}$.
2. $y_{\text{cm}} = (0 + 0 + 12)/6.0 = \mathbf{2.0 \text{ m}}$.

Problem 8 Non-uniform rod

1. $M = \int_0^2 3.0x \, dx = 1.5x^2 = \mathbf{6.0 \text{ kg}}$.
2. $\int x\lambda \, dx = \int_0^2 3.0x^2 \, dx = x^3 = 8.0$; $x_{\text{cm}} = 8.0/6.0 = \mathbf{1.3 \text{ m}}$ (2L/3).

Problem 9 Impulse from a graph

1. Triangle: $\frac{1}{2} \times 0.015 \times 400 = \mathbf{3.0 \text{ N}\cdot\text{s}}$.
2. $v = J/m = 3.0/0.060 = \mathbf{50 \text{ m/s}}$.

Problem 10 Cars locking together

1. $1200 \times 15 = 2000v$, so $v = \mathbf{9.0 \text{ m/s}}$.
2. $K_i = \frac{1}{2} \times 1200 \times 225 = 1.35 \times 10^5 \text{ J}$; $K_f = \frac{1}{2} \times 2000 \times 81 = 8.1 \times 10^4 \text{ J}$; loss = **54 kJ**.

Problem 11 Elastic collision on a track

$v_1 = (m_1 - m_2)v/(m_1 + m_2) = (0.20/0.40) \times 4.0 = \mathbf{2.0 \text{ m/s}}$; $v_2 = 2m_1v/(m_1 + m_2) = (0.60/0.40) \times 4.0 = \mathbf{6.0 \text{ m/s}}$. Check: momentum $1.2 \rightarrow 0.6 + 0.6 \checkmark$.

Problem 12 Exploding shell

1. $4.0 \times 20 = 3.0v$, so $v = \mathbf{27 \text{ m/s}}$.
2. At the same point where the unbroken shell would have landed, because gravity is the only external force.

Problem 13 Impulse mistake

1. Force $\times$ distance is *work* (energy), not impulse. Impulse needs force $\times$ time.
2. $J = 800 \times 0.0040 = \mathbf{3.2 \text{ N}\cdot\text{s}}$.

Problem 14 Time-varying force

1. $J = \int_0^2 6.0t^2 \, dt = 2.0t^3 = \mathbf{16 \text{ N}\cdot\text{s}}$.
2. $v = J/m = 16/4.0 = \mathbf{4.0 \text{ m/s}}$.

Problem 15 Semicircular plate

1. The strip has width $2\sqrt{(R^2 - y^2)}$ and thickness dy , so $dm = 2\sigma\sqrt{(R^2 - y^2)} \, dy$.
2. $M = \sigma\pi R^2/2$. $\int y \, dm = 2\sigma \int_0^R y\sqrt{(R^2 - y^2)} \, dy = 2\sigma \times R^3/3 = (2/3)\sigma R^3$, so $y_{\text{cm}} = (2\sigma R^3/3)/(\sigma\pi R^2/2) = \mathbf{4R/(3\pi)}$.

3. $y_{\text{cm}} = 4 \times 0.30 / (3\pi) = \mathbf{0.13 \text{ m}}$.

Problem 16 Density rising linearly

1. $M = \lambda_0 \int_0^L (1 + x/L) dx = \lambda_0(L + L/2) = 1.5\lambda_0 L$. $\int x \lambda dx = \lambda_0(L^2/2 + L^2/3) = 5\lambda_0 L^2/6$. Ratio: $(5/6)/(3/2) L = \mathbf{5L/9}$.

2. $x_{\text{cm}} = 5 \times 0.90/9 = \mathbf{0.50 \text{ m}}$.

Problem 17 Rocket burn

1. $\Delta v = v_e \ln(m_0/m) = 2000 \times \ln(500/150) = 2000 \times 1.204 = \mathbf{2.4 \times 10^3 \text{ m/s}}$.

2. Thrust $= v_e |dm/dt| = 2000 \times 5.0 = \mathbf{1.0 \times 10^4 \text{ N}}$.

Problem 18 Sand on a belt

1. $F = v dm/dt = 0.50 \times 8.0 = \mathbf{4.0 \text{ N}}$.

2. $P = Fv = 4.0 \times 0.50 = \mathbf{2.0 \text{ W}}$.

3. $dK/dt = \frac{1}{2}(dm/dt)v^2 = \frac{1}{2} \times 8.0 \times 0.25 = \mathbf{1.0 \text{ W}}$. The other 1.0 W becomes thermal energy as the sand slips against the belt.

Problem 19 Ballistic pendulum

1. Energy conservation: $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.12} = 1.53 \text{ m/s}$.

2. Momentum: $0.010v_0 = 2.010 \times 1.534$, so $v_0 = \mathbf{3.1 \times 10^2 \text{ m/s}}$.

Problem 20 Billiard balls at an angle

1. $x: 4.0 = 3.2 \times 0.80 + v_x$, so $v_x = 1.44 \text{ m/s}$. $y: 0 = 3.2 \times 0.60 + v_y$, so $v_y = -1.92 \text{ m/s}$.

2. $v = \sqrt{(1.44^2 + 1.92^2)} = \mathbf{2.4 \text{ m/s}}$, at 53° below the original direction.

3. $K_i = \frac{1}{2} \times 0.17 \times 16 = 1.36 \text{ J}$; $K_f = \frac{1}{2} \times 0.17 \times (10.24 + 5.76) = 1.36 \text{ J}$, so the collision **was elastic**.