

Centripetal acceleration and gravity in orbit

AP Physics 1 (algebra-based) · Lesson

Quick answer: Uniform circular motion is travel at constant speed along a circle. The velocity direction keeps changing, so the object accelerates toward the centre with $a = v^2/r$. The net force causing this is $m v^2/r$. In an orbit, gravity $F = G m_1 m_2 / r^2$ supplies that net force.

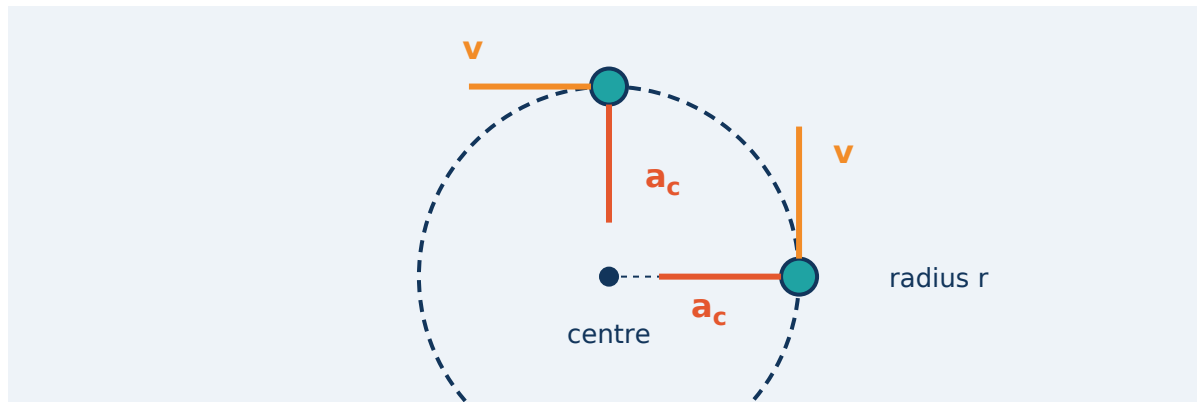
Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 2

What is uniform circular motion?

An object in uniform circular motion travels around a circle at constant speed. Its velocity is not constant, because the direction changes at every instant. A changing velocity means acceleration, so the object accelerates even though its speed never changes.

Definition

Centripetal acceleration is the acceleration of an object moving on a circular path. It points toward the centre of the circle and is perpendicular to the velocity, so it changes the direction of the motion but not the speed.



Top view of uniform circular motion. At every point the velocity v is tangent to the circle and the acceleration a_c points to the centre.

How do you calculate centripetal acceleration and force?

Formula

$a_c = v^2 / r = 4\pi^2 r / T^2$ and $\Sigma F = m a_c = m v^2 / r$. Here a_c is centripetal acceleration (m/s^2), v is speed (m/s), r is the radius (m), T is the period (s), m is mass (kg) and ΣF is the net force toward the centre (N).

The phrase centripetal force names a role, not a new kind of force. Add up the real forces that point toward the centre, subtract those that point away, and the result must equal $m v^2 / r$. Never draw an arrow labelled centripetal force on a free-body diagram.

Worked example 1

Kiran in Chennai whirls a 0.40 kg stone on a 0.75 m string in a horizontal circle on a smooth table, two revolutions every second. **Find the tension.**

Period $T = 1 \div 2 = 0.50 \text{ s}$. Speed $v = 2\pi r / T = 2\pi(0.75) \div 0.50 = 9.42 \text{ m/s}$.

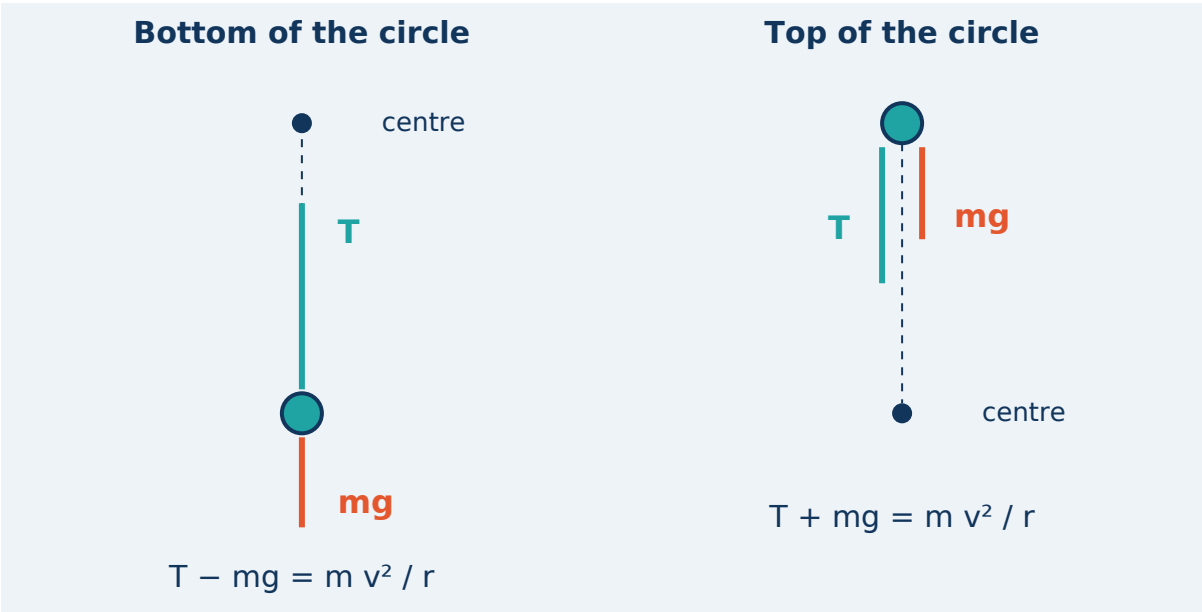
$a_c = v^2 / r = (9.42)^2 \div 0.75 = 118 \text{ m/s}^2$.

Tension $= m a_c = 0.40 \times 118 = \mathbf{47 \text{ N}}$, directed toward the centre.

What supplies the centripetal force?

Each circular motion has its own source of inward force. Identify it before you write any equation.

Situation	Real force that supplies the centripetal force
Stone whirled on a string	Tension in the string
Car turning on a flat road	Static friction between tyres and road
Moon orbiting the Earth	Gravitational attraction of the Earth
Rider pinned to the wall of a spinning drum	Normal force from the wall
Bucket at the top of a vertical circle	Weight plus the normal force or tension, both pointing down



Vertical circle. The net force must point to the centre, so the tension is largest at the bottom and smallest at the top.

In a vertical circle the weight always points down, but the centre can be above or below the object. Choose the direction toward the centre as positive and the equations follow directly. At the very top, the least speed that keeps a string taut or water in a bucket is found by setting the tension or normal force to zero, which gives $v = \sqrt{g r}$.

Common mistake

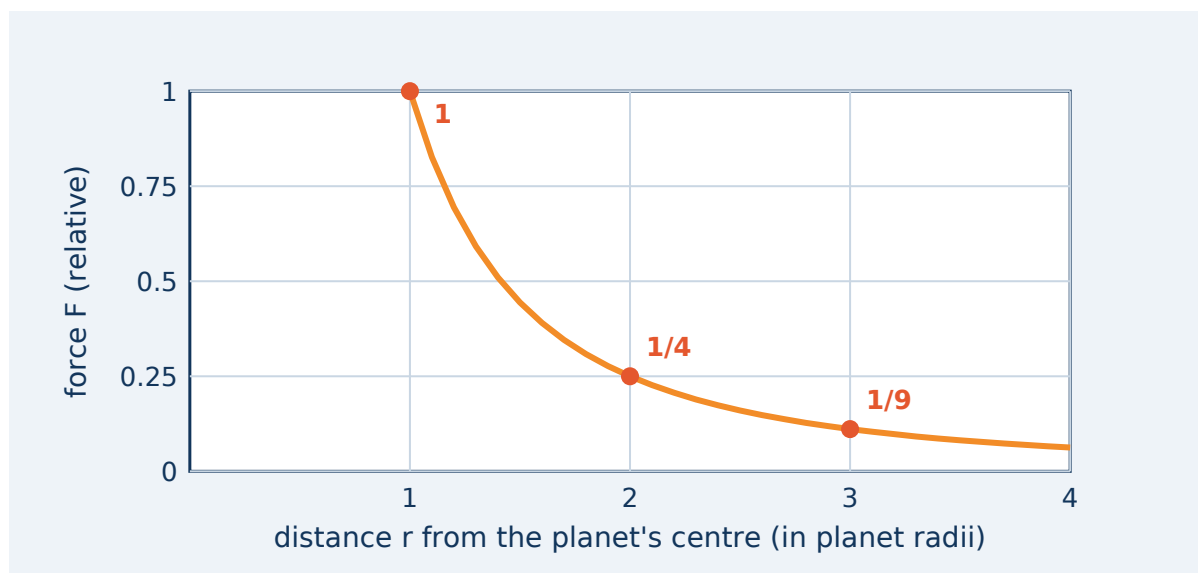
Students say that a centrifugal force pushes the object outward. Nothing pushes outward. The object tends to continue in a straight line, so if the inward force disappears it moves along the tangent.

How does universal gravitation work?

Key law

Every pair of masses attracts with a force $F = G m_1 m_2 / r^2$, where $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ and r is the distance between the centres. The two forces are equal in size and opposite in direction, as Newton's third law requires.

Dividing the force on a small mass by that mass gives the gravitational field strength $g = G M / r^2$. At the surface of the Earth this is about 9.81 N/kg , and it falls to one quarter at a distance of two Earth radii from the centre.



Gravity follows an inverse-square law: doubling the distance cuts the force to one quarter.

How fast does a satellite move in a circular orbit?

For a satellite, gravity is the only force, so it supplies the whole centripetal force: $G M m / r^2 = m v^2 / r$. The satellite mass cancels, so the orbit speed does not depend on it.

Formula

$v = \sqrt{G M / r}$ and $T^2 = 4\pi^2 r^3 / (G M)$. Here M is the mass of the central body (kg), r is the orbit radius measured from the centre (m) and T is the period (s).

Worked example 2

A survey satellite orbits 600 km above the Earth's surface. Use $G M = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$ and Earth radius $6.37 \times 10^6 \text{ m}$. **Find its speed and period.**

$r = 6.37 \times 10^6 + 6.0 \times 10^5 = 6.97 \times 10^6 \text{ m}$.

$v = \sqrt{G M / r} = \sqrt{3.98 \times 10^{14} \div 6.97 \times 10^6} = 7.56 \times 10^3 \text{ m/s}$.

$$T = 2\pi r / v = 2\pi(6.97 \times 10^6) \div 7.56 \times 10^3 = 5.79 \times 10^3 \text{ s} = \mathbf{96.6 \text{ min.}}$$

A larger orbit therefore means a lower speed and a longer period. Astronauts float because they and the station fall together with the same acceleration, not because gravity has vanished.

Lina's tip

Start every circular-motion problem by sketching the object and the centre, then write Newton's second law along the line to the centre. Only after that decide which numbers to insert.

Key points to remember

- Centripetal acceleration: $a = v^2/r = 4\pi^2 r/T^2$, directed toward the centre (m/s^2).
- Net force toward the centre: $\Sigma F = m v^2/r$, not a separate force.
- Gravity: $F = G m_1 m_2 / r^2$, with $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.
- Circular orbit: $v = \sqrt{G M / r}$ and $T^2 = 4\pi^2 r^3 / (G M)$.

Centripetal acceleration and gravity in orbit: frequently asked questions

Why does an object moving at constant speed on a circle accelerate?

Acceleration means a change in velocity, and velocity includes direction. On a circle the direction changes continuously, so the velocity changes and the acceleration points toward the centre.

Is centripetal force a separate force?

No. It is the name for the net force toward the centre, made up of real forces such as tension, friction, gravity or the normal force. It never appears as an extra arrow on a free-body diagram.

What happens if the string breaks?

With no inward force the object moves in a straight line along the tangent to the circle at the point of release, at the speed it had at that moment.

How does the gravitational force change with distance?

It follows an inverse-square law. Double the distance between the centres and the force becomes one quarter; triple it and the force becomes one ninth.

Why are astronauts weightless in orbit?

Gravity at 400 km is still about 89 % of its surface value. The astronauts and the station are in free fall together, so there is no contact force between them, which feels like weightlessness.