

# Centripetal acceleration and gravity in orbit

## AP Physics 1 (algebra-based) · Worksheet answers

Full methods are shown for every problem. Check the direction of the net force first, then the formula, then the units. Final results are in bold and rounded to sensible significant figures.

**Level:** AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 2

### Problem 1 Direction check

1. Toward the centre of the circle.
2. Also toward the centre, because net force and acceleration have the same direction.
3. The net force is perpendicular to the velocity, so it has no component along the motion and cannot change the speed; it changes only the direction.

**Acceleration and net force both point to the centre; the speed is unchanged.**

### Problem 2 Track cycling

1.  $a_c = v^2 / r = 7.2^2 \div 24 = 51.84 \div 24 = \mathbf{2.2 \text{ m/s}^2}$ .
2.  $\Sigma F = m a_c = 80 \times 2.16 = \mathbf{1.7 \times 10^2 \text{ N}}$  toward the centre.
3.  $T = 2\pi r / v = 2\pi(24) \div 7.2 = \mathbf{21 \text{ s}}$ .

### Problem 3 Record player

1.  $T = 60 \div 45 = 1.33 \text{ s}$ .
2.  $v = 2\pi r / T = 2\pi(0.12) \div 1.333 = 0.565 \text{ m/s}$ .
3.  $a_c = v^2 / r = 0.565^2 \div 0.12 = \mathbf{2.7 \text{ m/s}^2}$ .

### Problem 4 Scaling the pull

$$F \propto m_1 m_2 / r^2.$$

1. Halving  $r$  multiplies  $F$  by  $2^2 = 4$ , giving **4F**.
2. Factor  $3 \div 3^2 = 1/3$ , giving **F/3**.
3. Factor  $(2 \times 2) \div 2^2 = 1$ , giving **F** (unchanged).

### Problem 5 Two cargo ships

1.  $F = G m_1 m_2 / r^2 = 6.67 \times 10^{-11} \times (5.0 \times 10^7)^2 \div 200^2 = 6.67 \times 10^{-11} \times 6.25 \times 10^{10} = \mathbf{4.2 \text{ N}}$ .
2. The force is tiny compared with the water resistance and the mooring forces on ships of this mass, so friction and ropes easily hold them in place.

### Problem 6 Real forces only

1. Static friction between the coin and the turntable.
2. Tension in the string (with the horizontal component of tension if the string is not level).
3. The gravitational attraction of the Earth.
4. The normal force exerted by the wall.

**Friction, tension, gravity and the normal force respectively.**

### Problem 7 Coin on a turntable

1. Static friction.
2. Maximum friction  $\mu_s m g = m v^2 / r$ , so  $v = \sqrt{(\mu_s g r)} = \sqrt{(0.45 \times 9.81 \times 0.080)} = \mathbf{0.59 \text{ m/s}}$ .
3.  $\omega = v / r = 0.594 \div 0.080 = 7.43 \text{ rad/s}$ ;  $\text{rev/min} = 7.43 \times 60 \div 2\pi = \mathbf{71 \text{ rev/min}}$ .

### Problem 8 Rover on Mars

1.  $g = G M / R^2 = 6.67 \times 10^{-11} \times 6.42 \times 10^{23} \div (3.39 \times 10^6)^2 = 4.28 \times 10^{13} \div 1.149 \times 10^{13} = \mathbf{3.73 \text{ N/kg}}$ .
2.  $W = m g = 180 \times 3.73 = \mathbf{6.7 \times 10^2 \text{ N}}$ .
3. On Earth  $W = 180 \times 9.81 = 1766 \text{ N}$ , so the weight on Mars is  $670 \div 1766 = 0.38$  of its Earth value. The mass is the same on both planets.

### Problem 9 Orbit of Europa

1.  $v = \sqrt{(G M / r)} = \sqrt{(6.67 \times 10^{-11} \times 1.90 \times 10^{27} \div 6.71 \times 10^8)} = \sqrt{(1.889 \times 10^8)} = \mathbf{1.37 \times 10^4 \text{ m/s}}$ .
2.  $T = 2\pi r / v = 2\pi(6.71 \times 10^8) \div 1.374 \times 10^4 = 3.07 \times 10^5 \text{ s} \div 86\,400 = \mathbf{3.55 \text{ days}}$ .
3. They would not change, because the moon's mass cancels from  $G M m / r^2 = m v^2 / r$ .

### Problem 10 Swinging a bucket

1. At the limit the normal force on the water is zero, so  $m g = m v^2 / r$  and  $v = \sqrt{(g r)} = \sqrt{(9.81 \times 0.90)} = \mathbf{2.97 \text{ m/s}}$ .
2.  $T - m g = m v^2 / r$ , so  $T = m(g + v^2 / r) = 1.5 \times (9.81 + 25 \div 0.90) = 1.5 \times 37.6 = \mathbf{56 \text{ N}}$ .

### Problem 11 Reading a straight line

1.  $a_c = v^2 / r$  has the form  $y = (1 / r) x$  with fixed  $r$ , so  $a_c$  is proportional to  $v^2$ .
2. Gradient =  $2.0 \div 40 = 0.050 \text{ m}^{-1}$ , and gradient =  $1 / r$ , so  $r = \mathbf{20 \text{ m}}$ .
3.  $a_c = 8.0^2 \div 20 = \mathbf{3.2 \text{ m/s}^2}$ .

### Problem 12 Comparing two orbits

1.  $v \propto 1/\sqrt{r}$ , so  $v_B / v_A = 1/\sqrt{4} = \mathbf{1/2}$ .
2.  $T \propto r^{3/2}$ , so  $T_B / T_A = 4^{3/2} = \mathbf{8}$ .

3.  $a_c = G M / r^2 \propto 1/r^2$ , so the ratio is  $1/16 = \mathbf{0.0625}$ .

### Problem 13 Spot the slip

1. The speed must be squared: the force is  $m v^2 / r$ , not  $m v / r$ .
2.  $F = 900 \times 20^2 \div 50 = 900 \times 400 \div 50 = \mathbf{7200 \text{ N}}$ , twenty times larger than the student's value.

### Problem 14 Floating in orbit

1.  $r = 6.37 \times 10^6 + 4.0 \times 10^5 = 6.77 \times 10^6 \text{ m}$ , so  $g = G M / r^2 = 3.98 \times 10^{14} \div (6.77 \times 10^6)^2 = \mathbf{8.69 \text{ N/kg}}$ .
2.  $8.69 \div 9.81 = \mathbf{89 \%}$ .
3. Station and astronauts accelerate toward the Earth at the same rate (free fall), so no contact force acts between them and they feel weightless.

### Problem 15 Tractor on a farm track

1.  $v = \sqrt{(\mu_s g r)} = \sqrt{(0.60 \times 9.81 \times 35)} = \mathbf{14 \text{ m/s}}$ .
2.  $v = \sqrt{(0.25 \times 9.81 \times 35)} = \mathbf{9.3 \text{ m/s}}$ .
3. The most friction available is  $\mu_s m g$ . A smaller  $\mu_s$  means a smaller maximum inward force, and since the force needed is  $m v^2 / r$ , the speed must drop.

### Problem 16 Ferris wheel rider

1.  $v = 2\pi(12) \div 30 = 2.51 \text{ m/s}$ ;  $a_c = v^2 / r = 6.317 \div 12 = \mathbf{0.53 \text{ m/s}^2}$ .
2. Top (centre below):  $m g - N = m a_c$ , so  $N = 55(9.81 - 0.526) = \mathbf{5.1 \times 10^2 \text{ N}}$ .
3. Bottom (centre above):  $N - m g = m a_c$ , so  $N = 55(9.81 + 0.526) = \mathbf{5.7 \times 10^2 \text{ N}}$ .
4. At the top the net force toward the centre is downward, so the seat pushes up with less than the weight; at the bottom it must push up with more than the weight. The seat force is what she feels, so she feels lightest at the top.

### Problem 17 Satellite that stays overhead

1.  $G M m / r^2 = m v^2 / r$  with  $v = 2\pi r / T$  gives  $G M / r^2 = 4\pi^2 r / T^2$ , so  $T^2 = 4\pi^2 r^3 / (G M)$ .
2.  $r^3 = G M T^2 \div 4\pi^2 = 3.98 \times 10^{14} \times (86\,400)^2 \div 39.48 = 7.53 \times 10^{22} \text{ m}^3$ , so  $r = \mathbf{4.22 \times 10^7 \text{ m}}$ .
3. Height  $= 4.223 \times 10^7 - 6.37 \times 10^6 = \mathbf{3.59 \times 10^7 \text{ m}}$ ;  $v = 2\pi r / T = 2\pi(4.223 \times 10^7) \div 86\,400 = \mathbf{3.07 \times 10^3 \text{ m/s}}$ .

### Problem 18 Bigger planet, same mass

1. Smaller: the surface is farther from the centre, so the attraction is weaker.
2.  $g \propto M / R^2$ , so the ratio  $= 1 / 2^2 = \mathbf{1/4}$ , about 2.5 N/kg.
3.  $v = \sqrt{(G M / R)} \propto 1/\sqrt{R}$ , so the ratio is  $1/\sqrt{2} = \mathbf{0.71}$ .

4. The Earth needs the greater launch speed. Its surface field is four times stronger and its skimming-orbit speed is 1.41 times larger, so a rocket must supply more kinetic energy per kilogram to reach orbit. **Earth: field 4 times stronger, orbital speed 1.41 times larger.**

### Problem 19 Design: force against period

1. Independent: the tension (weight of hanging masses); dependent: period  $T$ ; controls: bung mass and radius.
2. Pass the string through a smooth tube with a hanging mass at the end; use a metre rule, a stopwatch timing 20 revolutions, a balance and a marker on the string to hold the radius.
3. From  $F = 4\pi^2 m r / T^2$ , plot  $F$  against  $1/T^2$ . The line is straight through the origin and the gradient equals  $4\pi^2 m r = 4\pi^2 \times 0.200 \times 0.50 = \mathbf{3.9 \text{ N s}^2}$ .
4. Timing error: time many revolutions and repeat. Drift of the radius or a sagging string: watch the marker and swing faster so the string stays level.

### Problem 20 Double asteroid

1. Each moves on a circle of radius  $d / 2 = \mathbf{1.0 \text{ km}}$  about the midpoint.
2. The force between them is  $G m^2 / d^2$ . This supplies  $m v^2 / (d/2)$ , so  $G m^2 / d^2 = 2 m v^2 / d$ , giving  $v^2 = G m / (2d)$ .
3.  $v = \sqrt{(6.67 \times 10^{-11} \times 8.0 \times 10^{11} \div 4000)} = \sqrt{(0.01334)} = \mathbf{0.12 \text{ m/s}}$ .  $T = 2\pi(1000) \div 0.1155 = 5.44 \times 10^4 \text{ s} = \mathbf{15 \text{ h}}$ .