

Angular velocity, centripetal force and circular paths

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: Circular motion at constant speed involves a constant angular velocity $\omega = 2\pi/T$ and a centripetal acceleration $a = v^2/r = \omega^2 r$ directed towards the centre. The resultant force that causes it, $F = mv^2/r$, is supplied by tension, friction, gravity or a normal reaction, and does no work.

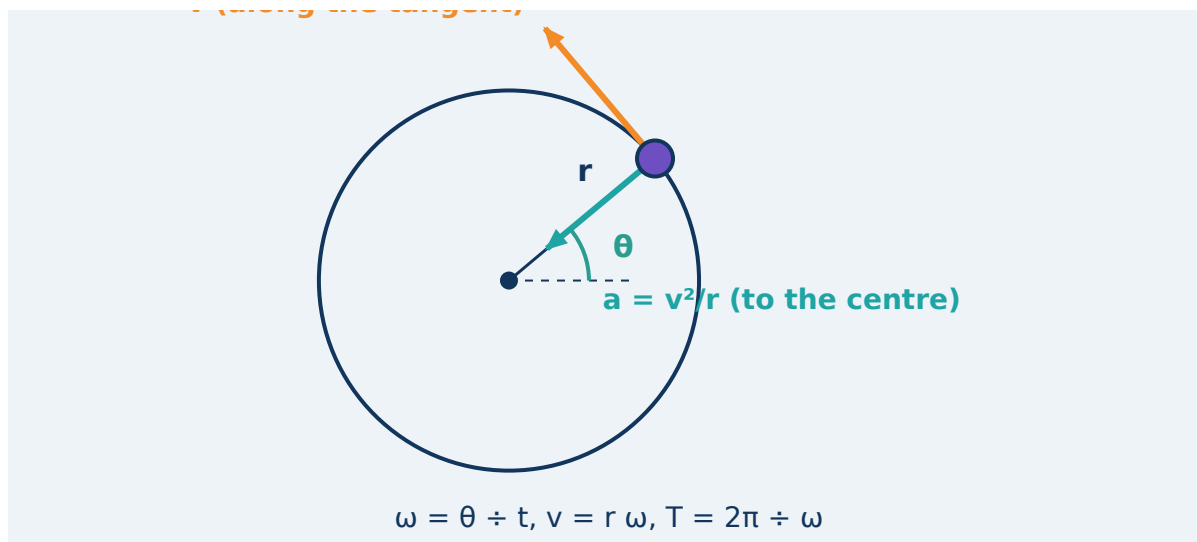
Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.6

How do you describe motion in a circle?

An object going round a circle keeps changing direction, even when its speed is constant. You describe how fast it turns with the **angular velocity** ω , measured in radians per second. One full turn is 2π radians, so $\omega = 2\pi \div T = 2\pi f$, where T is the period and f the frequency.

Definition

Angular velocity ω is the angle turned per unit time (rad/s). The **linear speed** along the circle is $v = r \omega$, where r is the radius. One radian is the angle at the centre of a circle subtended by an arc equal in length to the radius, so $180^\circ = \pi$ rad.



In uniform circular motion the speed is constant but the velocity changes direction, so there is an acceleration towards the centre.

Worked example 1

A merry-go-round in Kumasi makes one turn in 8.0 s. A child sits 2.5 m from the axis.

$$\omega = 2\pi \div T = 6.283 \div 8.0 = 0.785 \text{ rad/s.}$$

$$v = r \omega = 2.5 \times 0.785 = \mathbf{2.0 \text{ m/s.}}$$

Why is there an acceleration in uniform circular motion?

Velocity is a vector. When its direction changes, the object accelerates, even if the speed does not change. The acceleration points towards the centre. It is called centripetal ("centre-seeking") acceleration, and the resultant force that causes it is the **centripetal force**.

Formulae

$$a = v^2 \div r = \omega^2 r$$

$$F = m v^2 \div r = m \omega^2 r$$

a = centripetal acceleration (m/s^2), F = resultant force towards the centre (N), m = mass (kg), v = speed (m/s), ω = angular velocity (rad/s), r = radius (m).

Centripetal force is not a new kind of force. It is the name for whichever real force, or combination, points to the centre: tension in a string, friction on a tyre, gravity on a planet, or a normal reaction. Because the force is always at right angles to the velocity, it does no work and the speed stays constant.

Worked example 2

A stone of mass 0.30 kg is whirled on a string in a horizontal circle of radius 0.80 m at 4.0 m/s.

$$a = v^2 \div r = 16 \div 0.80 = 20 \text{ m/s}^2.$$

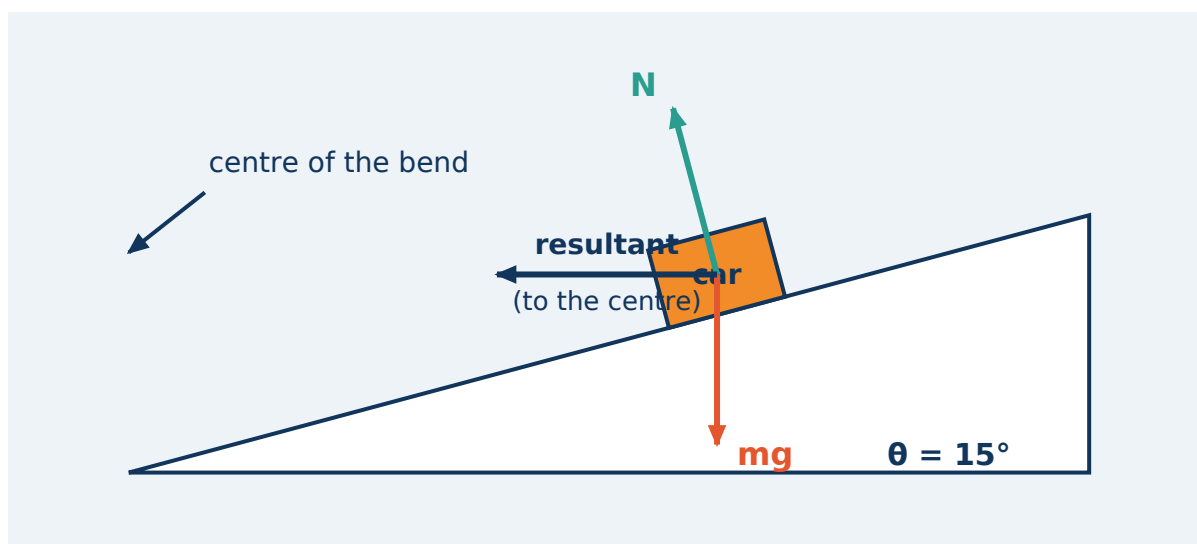
$$F = m a = 0.30 \times 20 = \mathbf{6.0 \text{ N}}, \text{ which is the tension in the string.}$$

Situation	Force that provides the centripetal force
Car on a flat bend	friction between the tyres and the road
Moon orbiting the Earth	gravitational attraction
Stone on a string	tension in the string
Rider pressed on the wall of a spinning drum	normal reaction of the wall

How do flat and banked bends work?

On a flat bend the friction at the tyres provides the centripetal force. The greatest value is $\mu m g$, so the car just keeps its circle when $\mu m g = m v^2 \div r$. The mass cancels, giving $v_{\text{max}} = \sqrt{(\mu g r)}$.

On a banked track the normal reaction N has a horizontal component $N \sin \theta$ that points to the centre. With no friction, $N \cos \theta = m g$ and $N \sin \theta = m v^2 \div r$, so **$\tan \theta = v^2 \div (r g)$** .



On a banked bend, the horizontal part of the normal reaction supplies the centripetal force.

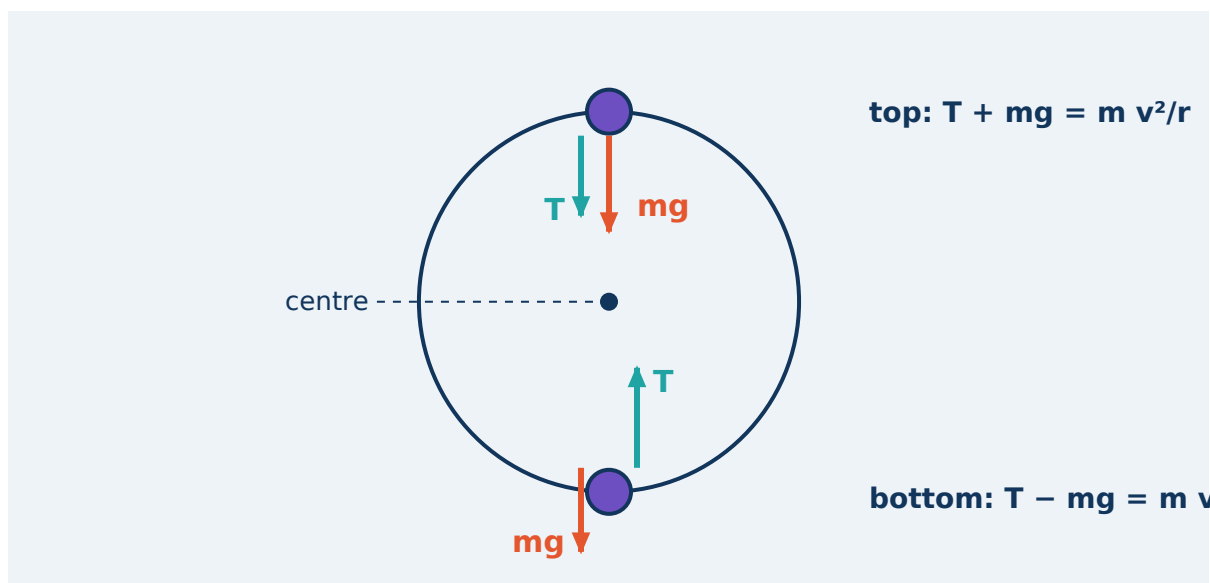
Worked example 3

A frictionless banked bend has radius 80 m and a design speed of 20 m/s.

$\tan \theta = v^2 \div (r g) = 400 \div (80 \times 9.81) = 0.510$, so $\theta = 27^\circ$.

What changes in a vertical circle?

In a vertical circle the weight sometimes helps and sometimes opposes the centripetal force, so the speed and the tension vary. At the bottom, $T - m g = m v^2 \div r$. At the top, $T + m g = m v^2 \div r$.



In a vertical circle the tension changes: it is smallest at the top and largest at the bottom.

Worked example 4

Wanjiru swings a bucket of water in a vertical circle of radius 0.80 m. The water stays in the bucket if the normal reaction at the top is not negative. At the limit $T = 0$, so $m g = m v^2 \div r$.

$v_{\min} = \sqrt{(g r)} = \sqrt{(9.81 \times 0.80)} = 2.8 \text{ m/s}$.

Common mistake

Many students draw an outward "centrifugal force" on the object. No such real force acts on it. The object tends to move in a straight line, and the inward resultant force bends its path. Draw only real forces, then say which direction is the net force.

Lina's tip

Start every circular motion problem by drawing the real forces, then pick the direction to the centre as positive. Write "net force towards the centre = $m v^2 \div r$ " and fill in the forces with their signs.

Key points to remember

- Angular velocity $\omega = 2\pi/T = 2\pi f$ in rad/s; linear speed $v = r \omega$.
- Centripetal acceleration $a = v^2/r = \omega^2 r$, directed towards the centre.
- Centripetal force $F = m v^2/r = m \omega^2 r$; it is a real force or a resultant, not an extra force.
- Flat bend: $v_{\max} = \sqrt{\mu g r}$; frictionless banked bend: $\tan \theta = v^2/(r g)$.
- Vertical circle: $T - m g = m v^2/r$ at the bottom, $T + m g = m v^2/r$ at the top.

Angular velocity, centripetal force and circular paths: frequently asked questions

What is the difference between angular velocity and linear speed?

Angular velocity ω is the angle turned per second in rad/s, the same for every point on a rotating disc. Linear speed $v = r \omega$ depends on the distance from the axis, so points farther out move faster.

Why does a satellite in a circular orbit not speed up?

Gravity is always at right angles to the velocity, so it changes the direction but does no work. The kinetic energy and so the speed stay constant.

Is centrifugal force real?

No real force pushes an object outward. What you feel in a turning car is your body tending to continue in a straight line, while the seat and door push you inward as the real centripetal force.

How do you convert revolutions per minute to rad/s?

Multiply by 2π to get radians per minute, then divide by 60. For example, 120 rpm is $120 \times 2\pi \div 60 = 12.6$ rad/s.

Why do roads have banked bends?

Banking lets the normal reaction supply part of the centripetal force. At the design speed no friction is needed at all, so the bend is safe even on ice or when it is wet.