

Angular velocity, centripetal force and circular paths

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit. If your answer is different, check that you converted rpm to rad/s, used radians for angular quantities and chose the correct direction for the net force before you substitute.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** A Level 3.6

Problem 1 Key relationships

1. $\omega = 2\pi \div T$.

2. $v = r \omega$.

3. $a = v^2 \div r = \omega^2 r$.

4. $F = m v^2 \div r$.

$\omega = 2\pi/T$, $v = r\omega$, $a = v^2/r = \omega^2 r$, $F = mv^2/r$.

Problem 2 Angles and rates

1. $180^\circ = \pi \text{ rad} = 3.14 \text{ rad}$.

2. $120 \times 2\pi \div 60 = 12.6 \text{ rad/s}$.

3. $\omega = 2\pi f = 2\pi \times 0.50 = 3.14 \text{ rad/s}$.

4. 120 rpm is 2.0 revolutions per second, so $T = 1 \div 2.0 = 0.50 \text{ s}$.

$\pi \text{ rad}$; 12.6 rad/s ; 3.14 rad/s ; 0.50 s .

Problem 3 Fan blade tip

1. $\omega = 1500 \times 2\pi \div 60 = 157 \text{ rad/s}$.

2. $v = r \omega = 0.18 \times 157.1 = 28.3 \text{ m/s}$.

$\omega = 157 \text{ rad/s}$; $v = 28 \text{ m/s}$.

Problem 4 Rotation of the Earth

1. $\omega = 2\pi \div 86\,400 = 7.27 \times 10^{-5} \text{ rad/s}$.

2. $v = r \omega = 6.37 \times 10^6 \times 7.27 \times 10^{-5} = 463 \text{ m/s}$.

$\omega = 7.27 \times 10^{-5} \text{ rad/s}$; $v = 463 \text{ m/s}$.

Problem 5 Toy on a string

1. $a = v^2 \div r = 9.0 \div 0.60 = 15 \text{ m/s}^2$.

2. $T = m a = 0.25 \times 15 = 3.75 \text{ N}$.

$a = 15 \text{ m/s}^2$; $T = 3.8 \text{ N}$.

Problem 6 What supplies the force

1. Friction between the tyres and the road.
2. Gravitational attraction of the Earth.
3. Tension in the string.
4. Normal reaction of the wall.

friction; gravity; tension; normal reaction.

Problem 7 Roundabout on a bicycle

1. $F = m v^2 \div r = 70 \times 36 \div 15 = 168 \text{ N}$.
2. $\mu m g \geq m v^2 \div r$, so $\mu = v^2 \div (r g) = 36 \div (15 \times 9.81) = 0.245$.
3. $v = \sqrt{(\mu g r)} = \sqrt{(0.20 \times 9.81 \times 15)} = \sqrt{29.43} = 5.4 \text{ m/s}$.

168 N; $\mu \geq 0.24$; 5.4 m/s.

Problem 8 Maximum speed on a bend

1. $v = \sqrt{(\mu g r)} = \sqrt{(0.65 \times 9.81 \times 45)} = \sqrt{287} = 17 \text{ m/s}$.
2. $v = \sqrt{(0.25 \times 9.81 \times 45)} = \sqrt{110} = 10.5 \text{ m/s}$.
3. The friction limit $\mu m g$ and the force needed $m v^2 \div r$ both contain m , so it cancels.

17 m/s; 10.5 m/s.

Problem 9 Whirling a bung

1. $F \div v^2 = 1.6 \div 4 = 0.40$, $3.6 \div 9 = 0.40$, $6.4 \div 16 = 0.40$, $10.0 \div 25 = 0.40$, $14.4 \div 36 = 0.40 \text{ N s}^2/\text{m}^2$.
2. F is proportional to v^2 , so the graph is a straight line through the origin with gradient $m \div r = 0.40 \text{ kg/m}$.
3. $m = \text{gradient} \times r = 0.40 \times 0.50 = 0.20 \text{ kg}$.

gradient = 0.40 kg/m; $m = 0.20 \text{ kg}$.

Problem 10 String breaks

1. Path P, along the tangent in the direction of motion.
2. When the string breaks, there is no resultant force, so the ball continues with its velocity at that instant: a straight line at constant speed.
3. The force was directed towards the centre of the circle (along the string).

path P.

Problem 11 Rotor ride

1. N is the only force towards the centre, so $N = m v^2 \div r$.
2. Friction balances weight: $\mu N = m g$, so $\mu m v^2 \div r = m g$ and $v = \sqrt{(g r \div \mu)} = \sqrt{(9.81 \times$

$$3.0 \div 0.40) = 8.6 \text{ m/s.}$$

$$3. \omega = v \div r = 2.86 \text{ rad/s, so rpm} = 2.86 \times 60 \div 2\pi = 27 \text{ rpm.}$$

$$\mathbf{v = 8.6 \text{ m/s; 27 rpm.}}$$

Problem 12 Hammer throw

$$1. F = m v^2 \div r = 7.26 \times 625 \div 1.8 = 2521 \text{ N.}$$

$$2. \text{Weight} = 7.26 \times 9.81 = 71.2 \text{ N, so the force is } 2521 \div 71.2 = 35 \text{ times the weight.}$$

$$\mathbf{F = 2.5 \times 10^3 \text{ N; 35 \times weight.}}$$

Problem 13 Ball at the bottom

$$1. T - m g = m v^2 \div r, \text{ so } T = 0.50 \times 9.81 + 0.50 \times 25 \div 1.2 = 4.905 + 10.42 = 15.3 \text{ N.}$$

$$2. \text{At the limit } T = 0, \text{ so } v = \sqrt{(g r)} = \sqrt{(9.81 \times 1.2)} = 3.4 \text{ m/s.}$$

$$\mathbf{T = 15 \text{ N; } v_{\min} = 3.4 \text{ m/s.}}$$

Problem 14 Railway curve

$$1. N \cos \theta = m g \text{ and } N \sin \theta = m v^2 \div r. \text{ Dividing gives } \tan \theta = v^2 \div (r g).$$

$$2. \tan \theta = 900 \div (900 \times 9.81) = 0.102, \text{ so } \theta = 5.8^\circ.$$

$$\mathbf{\theta = 5.8^\circ.}$$

Problem 15 Centrifugal force myth

1. The passenger tends to move in a straight line. The car turns inward, so the door moves into the passenger and pushes inward on the passenger, who experiences this as being pushed outward.

$$2. F = m v^2 \div r = 80 \times 144 \div 30 = 384 \text{ N.}$$

$$\mathbf{F = 384 \text{ N inward.}}$$

Problem 16 Banked bend with a car

$$1. v = \sqrt{(r g \tan \theta)} = \sqrt{(100 \times 9.81 \times \tan 15^\circ)} = \sqrt{(981 \times 0.2679)} = \sqrt{262.9} = 16.2 \text{ m/s.}$$

$$2. N = m g \div \cos \theta = 1200 \times 9.81 \div 0.9659 = 12\,190 \text{ N.}$$

$$3. N \sin \theta = 12\,190 \times 0.2588 = 3155 \text{ N. } m v^2 \div r = 1200 \times 262.9 \div 100 = 3155 \text{ N. They are equal.}$$

$$\mathbf{v = 16 \text{ m/s; } N = 1.22 \times 10^4 \text{ N.}}$$

Problem 17 Conical pendulum

$$1. r = L \sin \theta = 0.80 \times 0.50 = 0.40 \text{ m.}$$

$$2. \text{Vertically } T \cos \theta = m g, \text{ so } T = 0.15 \times 9.81 \div 0.866 = 1.70 \text{ N.}$$

$$3. \text{Horizontally } T \sin \theta = m \omega^2 r, \text{ so } \omega^2 = 1.70 \times 0.50 \div (0.15 \times 0.40) = 14.2 \text{ and } \omega = 3.76 \text{ rad/s. } T_{\text{period}} = 2\pi \div 3.76 = 1.67 \text{ s.}$$

$$\mathbf{T = 1.70 \text{ N; } \omega = 3.8 \text{ rad/s; period} = 1.7 \text{ s.}}$$

Problem 18 Loop-the-loop

1. At the top $N = 0$, so $v = \sqrt{g r} = \sqrt{9.81 \times 8.0} = 8.86 \text{ m/s}$.
2. The top is $2r = 16 \text{ m}$ above the bottom: $\frac{1}{2} v_b^2 = \frac{1}{2} v_t^2 + g (2 r)$, so $v_b^2 = 78.5 + 4 \times 9.81 \times 8.0 = 392$ and $v_b = 19.8 \text{ m/s}$.
3. $N - m g = m v^2 \div r$, so $N = 60 \times 9.81 + 60 \times 392 \div 8.0 = 589 + 2943 = 3532 \text{ N} = 6.0$ times the weight.
8.9 m/s; 19.8 m/s; N = 3.5 kN (6 mg).

Problem 19 Leaning motorcycle

1. $\tan \theta = 225 \div (40 \times 9.81) = 0.573$, so $\theta = 29.8^\circ$.
2. Friction must supply $m v^2 \div r$, with $N = m g$, so $\mu \geq \tan \theta = 0.57$.
3. $\tan \theta = 400 \div 392.4 = 1.019$, so $\theta = 45.5^\circ$. Raising the speed by a third needs a much larger lean and more friction ($\mu \geq 1.02$, which is above the value for most wet or dry roads).
 $\theta = 30^\circ$; $\mu \geq 0.57$; 46° .

Problem 20 Laboratory centrifuge

1. $\omega = 3000 \times 2\pi \div 60 = 314 \text{ rad/s}$.
2. $a = \omega^2 r = 314.2^2 \times 0.10 = 9870 \text{ m/s}^2$, which is $9870 \div 9.81 = 1.0 \times 10^3 \text{ g}$.
3. $F = m a = 0.012 \times 9870 = 118 \text{ N}$.
4. a is proportional to ω^2 , so the acceleration is four times as large.
 $\omega = 314 \text{ rad/s}$; $a = 9.9 \times 10^3 \text{ m/s}^2$ ($1.0 \times 10^3 \text{ g}$); $F = 118 \text{ N}$.