

Elastic and inelastic collisions with centre of mass

AP Physics 1 (algebra-based) · Lesson

Quick answer: In a collision, the total momentum of the colliding objects is the same before and after, provided no outside net force acts. Kinetic energy is conserved only in elastic collisions. The centre of mass of the system keeps moving at constant velocity throughout, because its velocity equals total momentum divided by total mass.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 4

What happens to momentum in a collision?

In a collision two objects push on each other for a very short time. By Newton's third law the forces are equal and opposite, so one object gains exactly the momentum the other loses. For the pair taken together, the total momentum before equals the total momentum after, as long as no outside net force acts on the pair during the contact.

Definition

A **collision** is an interaction in which objects exert large forces on each other for a short time. The **system** is the group of objects you choose to analyse. Momentum is conserved for a system when the net external force on it is zero, so $p_{\text{before}} = p_{\text{after}}$.

Collisions are sorted by what happens to kinetic energy. Momentum is conserved in all three types below.

Type	Momentum	Kinetic energy	After the contact
Elastic	conserved	conserved	objects bounce apart
Inelastic	conserved	some transformed to thermal energy, sound and deformation	objects separate or stay together
Perfectly inelastic	conserved	largest possible loss for the system	objects move off as one body

How do you solve a perfectly inelastic collision?

Formula

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

m = mass (kg), u = velocity before (m/s), v = common velocity after (m/s). Give each velocity a sign: positive one way, negative the other.

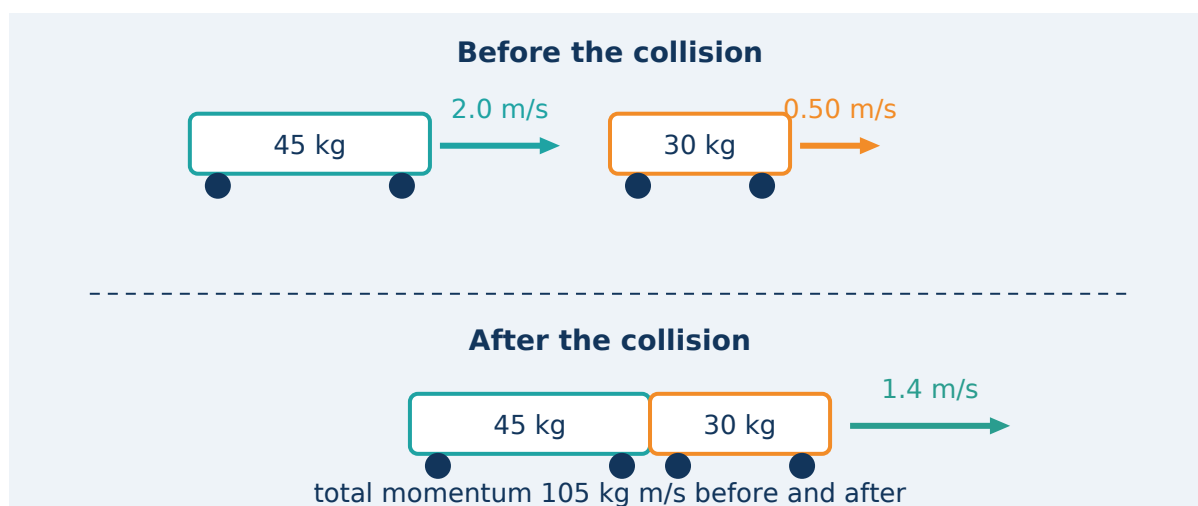
Worked example

At a market in Accra, Kofi pushes a 45 kg loaded trolley at 2.0 m/s. It catches a 30 kg trolley moving the same way at 0.50 m/s and the two hook together.

Momentum before = $45 \times 2.0 + 30 \times 0.50 = 105 \text{ kg m/s}$.

$v = 105 \div (45 + 30) = \mathbf{1.4 \text{ m/s}}$ in the original direction.

Kinetic energy before = $90 \text{ J} + 3.75 \text{ J} = 93.75 \text{ J}$. After = $\frac{1}{2} \times 75 \times 1.4^2 = 73.5 \text{ J}$. Loss = **20.25 J**, about 22 %.



The momentum arrows change but the total momentum stays 105 kg m/s.

What changes in an elastic collision?

In an elastic collision both momentum and kinetic energy are conserved. For two objects on a line with the second one initially at rest, solving the two equations gives the formulae below. A shortcut: the speed of approach equals the speed of separation.

Formula

$$v_1 = (m_1 - m_2) u_1 \div (m_1 + m_2) \text{ and } v_2 = 2 m_1 u_1 \div (m_1 + m_2)$$

valid when object 2 starts at rest. u_1 = incoming velocity (m/s); v_1, v_2 = velocities after (m/s).

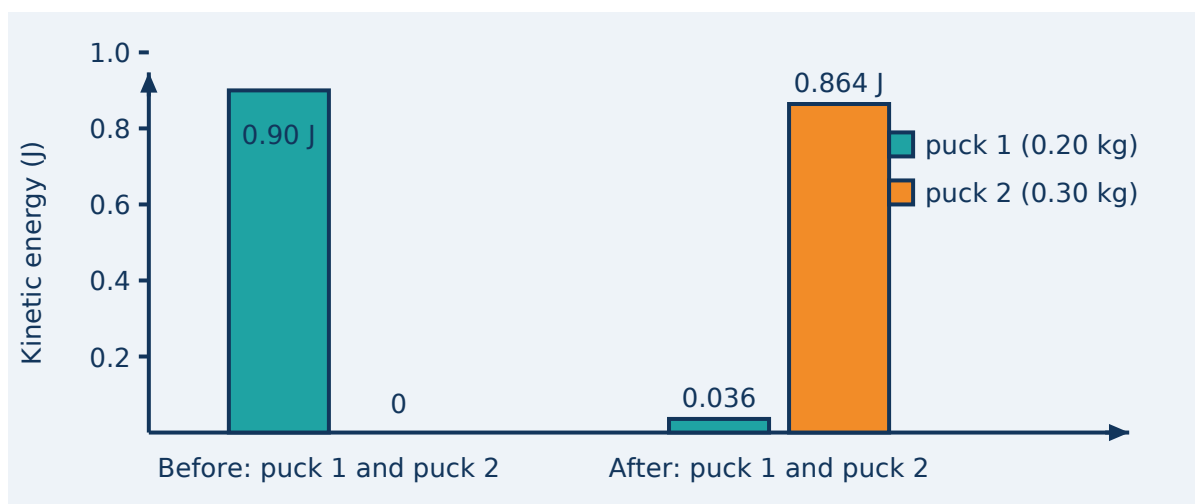
Worked example

On an air-hockey table in Pune, a 0.20 kg puck at 3.0 m/s hits a stationary 0.30 kg puck head-on. Assume the collision is elastic.

$$v_1 = (0.20 - 0.30) \times 3.0 \div 0.50 = \mathbf{-0.60 \text{ m/s}}$$
 (it rebounds).

$$v_2 = 2 \times 0.20 \times 3.0 \div 0.50 = \mathbf{2.4 \text{ m/s}}.$$

Check momentum: $0.20 \times 3.0 = 0.60 \text{ kg m/s}$ and $-0.12 + 0.72 = 0.60 \text{ kg m/s}$. Check energy: 0.90 J before and $0.036 \text{ J} + 0.864 \text{ J} = 0.90 \text{ J}$ after.



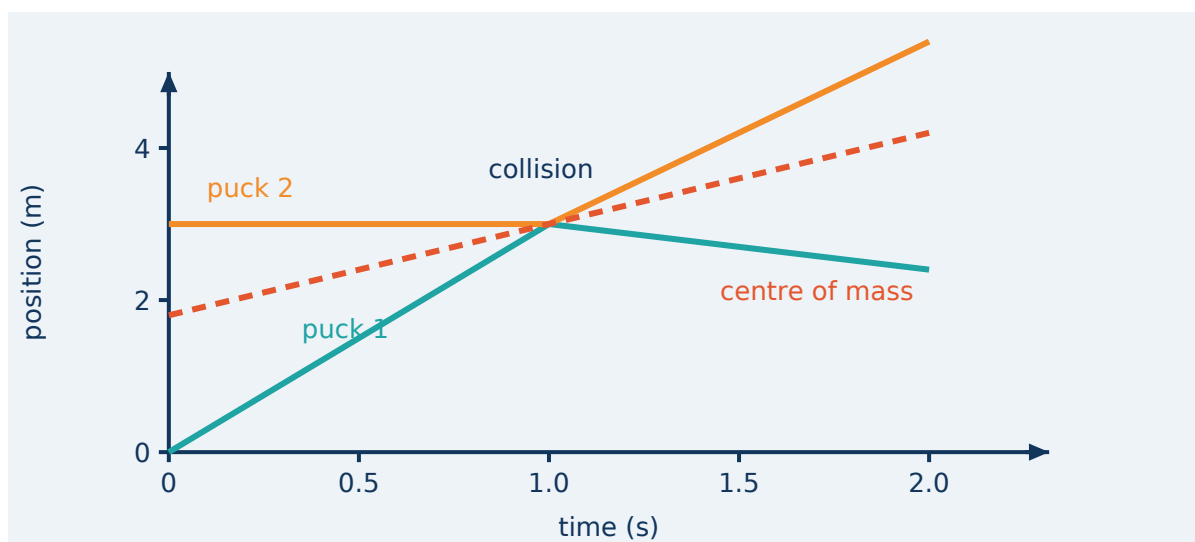
In an elastic collision the energy is shared out but the total is unchanged.

How does the centre of mass move during a collision?

Definition

The **centre of mass** is the single point whose position is the mass-weighted average of the positions of all parts of a system: $x_{cm} = (m_1x_1 + m_2x_2) \div (m_1 + m_2)$. Its velocity is $v_{cm} = (m_1v_1 + m_2v_2) \div (m_1 + m_2) = p_{total} \div M$.

Because v_{cm} equals total momentum divided by total mass, it cannot change when the net external force is zero. The collision rearranges the objects but the centre of mass carries on in a straight line at constant speed. In the puck example $v_{cm} = 0.60 \div 0.50 = 1.2$ m/s before and $0.60 \div 0.50 = 1.2$ m/s after.



Each puck changes its motion at the collision, but the centre of mass moves in one straight line.

Common mistake

Do not say that momentum is lost when kinetic energy is lost. In every isolated collision the total momentum is conserved. Only kinetic energy can be transformed, and only the perfectly inelastic case loses the most.

Lina's tip

Draw a before-and-after sketch first and write a sign on every velocity. Then write the momentum equation in words, and only after that put in numbers.

To classify a collision, calculate kinetic energy before and after. Equal totals mean elastic. A smaller total after means inelastic. Objects that move off together mean perfectly inelastic. A total that is larger after means energy was released, as in an explosion.

Key points to remember

- Total momentum is conserved when the net external force on the system is zero: $\Sigma p_{\text{before}} = \Sigma p_{\text{after}}$.
- Perfectly inelastic: $v = (m_1 u_1 + m_2 u_2) \div (m_1 + m_2)$, objects move together.
- Elastic: kinetic energy is conserved and the speed of approach equals the speed of separation.
- Centre of mass: $x_{\text{cm}} = \Sigma m x \div \Sigma m$ and $v_{\text{cm}} = \Sigma p \div \Sigma m$, constant in an isolated system.

Elastic and inelastic collisions with centre of mass: frequently asked questions

What is the difference between elastic and inelastic collisions?

In an elastic collision kinetic energy is conserved as well as momentum. In an inelastic collision some kinetic energy is transformed into thermal energy, sound or permanent deformation, while momentum is still conserved.

Is momentum conserved in an inelastic collision?

Yes. Momentum is conserved in every collision where the net external force on the system is negligible. Inelastic describes only the loss of kinetic energy.

Where does the lost kinetic energy go?

It is transformed into the internal energy of the objects and their surroundings: a small temperature rise, sound, and the work done bending or breaking materials.

Why does the centre of mass keep its velocity in a collision?

The forces between the objects are internal and cancel in pairs, so the net external force is zero. Since v_{cm} equals total momentum divided by total mass, it cannot change.

Can kinetic energy increase in a collision?

Not in a true collision, but it can in an explosion or a spring release. Stored chemical or elastic energy is transformed into kinetic energy while the total momentum stays the same.