

Elastic and inelastic collisions with centre of mass

AP Physics 1 (algebra-based) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit. Compare your own method as well as your final number. Where a problem asks for a written argument, the model answer is only one acceptable wording; the key physics ideas matter most.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 4

Problem 1 Sorting collisions

1. The clay lumps move off together: **perfectly inelastic**.
2. No kinetic energy is lost: **elastic**.
3. Kinetic energy is lost but the van still separates from the barrier: **inelastic**.

The quantity conserved in all three is the **total momentum of the system**, provided external forces are negligible.

Problem 2 Balancing point of two masses

$$x_{\text{cm}} = \Sigma mx \div \Sigma m.$$

1. $(3.0 \times 0 + 1.0 \times 0.80) \div 4.0 = \mathbf{0.20 \text{ m}}$.
2. $(2.0 \times (-1.0) + 6.0 \times 3.0) \div 8.0 = 16 \div 8.0 = \mathbf{2.0 \text{ m}}$.

Problem 3 Carts that stick

1. $p = 0.80 \times 1.5 = \mathbf{1.2 \text{ kg m/s}}$.
2. Momentum is conserved: $1.2 = (0.80 + 0.40) v$, so $v = 1.2 \div 1.20 = \mathbf{1.0 \text{ m/s}}$.

Problem 4 Choosing the system

1. Truck B exerts a force on A during the contact, and that force is external to A, so the momentum of A changes.
2. The forces between the trucks are equal and opposite (Newton's third law) and are internal to the pair. With no net external force, **the total momentum of the pair is conserved**.

Problem 5 Energy lost when carts stick

1. Before: $\frac{1}{2} \times 0.80 \times 1.5^2 = 0.90 \text{ J}$. After: $\frac{1}{2} \times 1.20 \times 1.0^2 = 0.60 \text{ J}$.
2. Loss = $0.90 - 0.60 = \mathbf{0.30 \text{ J}}$, which is $0.30 \div 0.90 = \mathbf{33 \%}$.

Problem 6 Centre of mass velocity

- $v_{cm} = (2.0 \times 4.0 + 3.0 \times (-2.0)) \div 5.0 = 2.0 \div 5.0 = \mathbf{+0.40 \text{ m/s}}$.
- It does not change: **+0.40 m/s**, because the net external force is zero.

Problem 7 Curling takeout

- $m_1 = m_2$, so $v_1 = 0 \times 2.0 \div 38 = \mathbf{0}$ and $v_2 = 2 \times 19 \times 2.0 \div 38 = \mathbf{2.0 \text{ m/s}}$. The moving stone stops and the other takes all the motion.
- Before: $19 \times 2.0 \div 38 = 1.0 \text{ m/s}$. After: $19 \times 2.0 \div 38 = \mathbf{1.0 \text{ m/s}}$, unchanged.

Problem 8 Crash test at the proving ground

- $1200 \times 20 = 2000 v$, so $v = 24\,000 \div 2000 = \mathbf{12 \text{ m/s}}$.
- Before: $\frac{1}{2} \times 1200 \times 20^2 = 2.4 \times 10^5 \text{ J}$. After: $\frac{1}{2} \times 2000 \times 12^2 = 1.44 \times 10^5 \text{ J}$. Loss = **$9.6 \times 10^4 \text{ J}$** .
- It becomes work deforming the metal, thermal energy and sound.

Problem 9 Find the unknown cart

- $0.50 \times 2.4 = (0.50 + m) \times 1.5$.
- $1.2 = 0.75 + 1.5 m$, so $m = 0.45 \div 1.5 = \mathbf{0.30 \text{ kg}}$.

Problem 10 Reading a velocity graph

- Cart A: 5.0 m/s before, 1.0 m/s after. Cart B: 0 before, 2.0 m/s after.
- Momentum: $2.0 \times 5.0 = 2.0 \times 1.0 + m \times 2.0$, so $m = 8.0 \div 2.0 = \mathbf{4.0 \text{ kg}}$.
- Before: $\frac{1}{2} \times 2.0 \times 5.0^2 = 25 \text{ J}$. After: $\frac{1}{2} \times 2.0 \times 1.0^2 + \frac{1}{2} \times 4.0 \times 2.0^2 = 1 + 8 = 9 \text{ J}$. Energy is lost, so the collision is **inelastic**.

Problem 11 Heavy hits light

- $v_1 = (2.0 - 1.0) \times 3.0 \div 3.0 = \mathbf{1.0 \text{ m/s}}$; $v_2 = 2 \times 2.0 \times 3.0 \div 3.0 = \mathbf{4.0 \text{ m/s}}$. Check: $6.0 = 2.0 + 4.0$.
- Approach: $3.0 - 0 = 3.0 \text{ m/s}$. Separation: $4.0 - 1.0 = 3.0 \text{ m/s}$. They are equal.

Problem 12 Head-on trolleys stop dead

- Take the first direction as positive: $20 \times 1.5 + 30 \times (-1.0) = 30 - 30 = 0$, so $v = \mathbf{0}$.
- Before: $\frac{1}{2} \times 20 \times 1.5^2 + \frac{1}{2} \times 30 \times 1.0^2 = 22.5 + 15 = 37.5 \text{ J}$. After: 0. Loss = **37.5 J** , all of it.

Problem 13 Equal and opposite impulses

- Larger cart: $0.80 \times (1.0 - 1.5) = \mathbf{-0.40 \text{ kg m/s}}$. Smaller cart: $0.40 \times (1.0 - 0) = \mathbf{+0.40}$

kg m/s. They are equal and opposite.

2. $F = \Delta p \div \Delta t = 0.40 \div 0.050 = \mathbf{8.0 \text{ N}}$ on each cart, in opposite directions.

Problem 14 Three masses on a light rod

1. $x_{\text{cm}} = (1.0 \times 0 + 2.0 \times 0.30 + 3.0 \times 0.90) \div 6.0 = 3.3 \div 6.0 = \mathbf{0.55 \text{ m}}$.

2. Place the pivot at the centre of mass, **0.55 m from the 1.0 kg end**.

Problem 15 Checking recorded data

1. Before: $0.50 \times 0.80 = 0.400 \text{ kg m/s}$. After: $0.50 \times 0.27 + 0.25 \times 1.06 = 0.135 + 0.265 = \mathbf{0.400 \text{ kg m/s}}$.

2. Before: $\frac{1}{2} \times 0.50 \times 0.80^2 = 0.160 \text{ J}$. After: $\frac{1}{2} \times 0.50 \times 0.27^2 + \frac{1}{2} \times 0.25 \times 1.06^2 = 0.0182 + 0.1405 = \mathbf{0.159 \text{ J}}$.

3. The energy differs by less than 1 %, so the collision is **elastic** within the measurement error.

Problem 16 Sticking to a heavy cart

1. $m_1 u = (m_1 + m_2) v$, so $\mathbf{v = m_1 u \div (m_1 + m_2)}$.

2. $\text{KE after} \div \text{KE before} = [\frac{1}{2} (m_1 + m_2) v^2] \div [\frac{1}{2} m_1 u^2] = (m_1 + m_2) m_1^2 u^2 \div [(m_1 + m_2)^2 m_1 u^2] = \mathbf{m_1 \div (m_1 + m_2)}$.

3. With $m_2 = 9 m_1$: $v = 5.0 \div 10 = \mathbf{0.50 \text{ m/s}}$. Retained fraction = $1 \div 10$, so **90 %** of the kinetic energy is lost.

Problem 17 Designing a test of momentum conservation

1. Measure the mass of each cart. Fit a card of known length to each cart; speed = card length $\div$ time to pass the photogate. Measure the speed of the moving cart before and the joined carts after. Repeat with different masses.

2. Calculate $p_{\text{before}} = m_1 u$ and $p_{\text{after}} = (m_1 + m_2) v$. Plot p_{after} against p_{before} : a straight line of **gradient 1 through the origin** within uncertainty supports conservation.

3. Friction on the track removes momentum, so p_{after} is slightly low. Tilt the track very slightly or use an air track, and take the speed just after the collision.

Problem 18 Timeline of a point collision

1. The carts approach at 4.0 m/s over a gap of 0.80 m: $t = 0.80 \div 4.0 = \mathbf{0.20 \text{ s}}$. Cart 1 is then at $0.20 + 2.0 \times 0.20 = \mathbf{0.60 \text{ m}}$.

2. $v_1 = [(1.5 - 0.5) \times 2.0 + 2 \times 0.5 \times (-2.0)] \div 2.0 = \mathbf{0}$; $v_2 = [(0.5 - 1.5) \times (-2.0) + 2 \times 1.5 \times 2.0] \div 2.0 = \mathbf{+4.0 \text{ m/s}}$. Check: momentum 2.0 kg m/s both times.

3. Method one: $v_{\text{cm}} = 2.0 \div 2.0 = 1.0 \text{ m/s}$ from $x_{\text{cm}} = (1.5 \times 0.20 + 0.50 \times 1.00) \div 2.0 = 0.40 \text{ m}$, so $x_{\text{cm}} = 0.40 + 1.0 \times 0.50 = \mathbf{0.90 \text{ m}}$. Method two: cart 1 at 0.60 m; cart 2 at $0.60 + 4.0 \times 0.30 = 1.80 \text{ m}$; $x_{\text{cm}} = (1.5 \times 0.60 + 0.50 \times 1.80) \div 2.0 = \mathbf{0.90 \text{ m}}$.

Problem 19 Partly inelastic bounce

1. Momentum before: $6.0 - 1.5 = 4.5$ kg m/s. After: $2.0 \times 0.50 + 1.0 \times v = 4.5$, so $v = +3.5$ m/s.
2. Before: $9.0 + 1.125 = 10.125$ J. After: $0.25 + 6.125 = 6.375$ J. Loss = **3.75 J**.
3. Approach 4.5 m/s, separation $3.5 - 0.50 = 3.0$ m/s. Ratio = **0.67**. A ratio of 1 means elastic and 0 means perfectly inelastic, so this collision is partly inelastic.

Problem 20 Evaluating a claim

1. Taking one direction as positive: $1500 \times 10 - 1500 \times 10 = 0$ before and 0 after. Kinetic energy before = $2 \times \frac{1}{2} \times 1500 \times 10^2 = \mathbf{1.5 \times 10^5 \text{ J}}$, all transformed.
2. The claim is wrong. For the system of both cars the total momentum was zero before and is zero after, so it is conserved. Each car lost its own momentum because the other car exerted a force on it, but the momentum of one car is the opposite of the other, so the total is unchanged. What disappeared was kinetic energy, which became deformation work, sound and thermal energy.