

Potential from fields and equipotentials

AP Physics C (Mechanics and E&M, calculus-based) · Lesson

Quick answer: Electric potential is potential energy per unit charge, $V = U/q$, measured in volts. The potential difference between two points is the negative line integral of the field, $\Delta V = -\int \mathbf{E} \cdot d\mathbf{l}$, and the field is the negative gradient of potential, $\mathbf{E} = -dV/dx$. Conductors in equilibrium are equipotentials with zero internal field.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 9

What is electric potential and how is it linked to the field?

A charge placed in an electric field has potential energy that depends on where it sits. Dividing that energy by the charge gives a property of the point itself, which is the electric potential. Because potential is a scalar, adding the contributions of many charges needs no vectors, and the field can always be recovered from it afterwards.

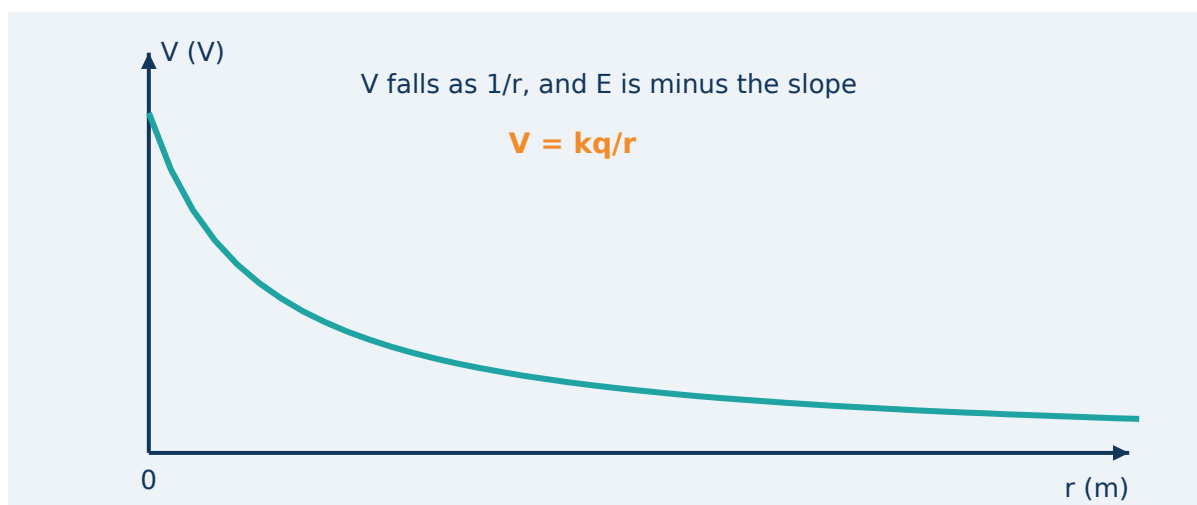
Definition

Electric potential at a point is the work per unit positive charge done by an external agent in bringing a small test charge from infinity to that point at constant speed: $V = U/q$. Its unit is the volt, $1 \text{ V} = 1 \text{ J/C}$.

Formula

$$V = (1/4\pi\epsilon_0) q/r = kq/r \quad \Delta V = V_b - V_a = -\int \mathbf{E} \cdot d\mathbf{l} \quad E_x = -dV/dx$$

V is potential (V), q is source charge (C), r is distance (m), E is field ($\text{V/m} = \text{N/C}$), $k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$. A charge q moved through ΔV changes its energy by $\Delta U = q\Delta V$.



Potential of a positive point charge: V is large near the charge and tends to zero at infinity.

How do you find potential by integrating the field?

Choose a path from the reference point to the point of interest and integrate $E \cdot dl$ along it. Since the electrostatic force is conservative, every path gives the same answer, so you may pick the easiest one. If you move against the field the potential rises, which is why V is highest next to a positive source.

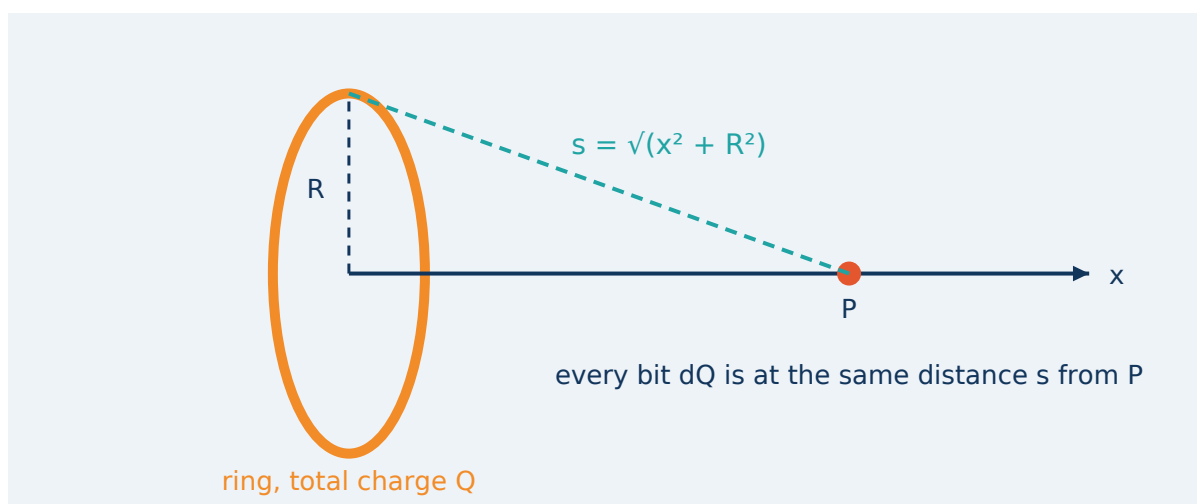
Worked example

Data: along the x -axis the field is $E = (600 \text{ V/m}^3)x^2$ in the $+x$ direction. **Find** $V(0.20 \text{ m}) - V(0)$.

Method: $\Delta V = -\int$ from 0 to 0.20 of $600x^2 dx = -200x^3$ evaluated at 0.20 m = -200×0.0080 .

Answer: $\Delta V = -1.6 \text{ V}$. The potential falls in the direction of the field.

For continuous charge, add the potentials of small pieces: $V = k \int dq/s$, where s is the distance from the piece to the point. This is easier than adding vector fields because every piece contributes with the same sign.



A ring on its axis: all pieces dQ lie at the same distance s , so $V = kQ/s$.

Worked example

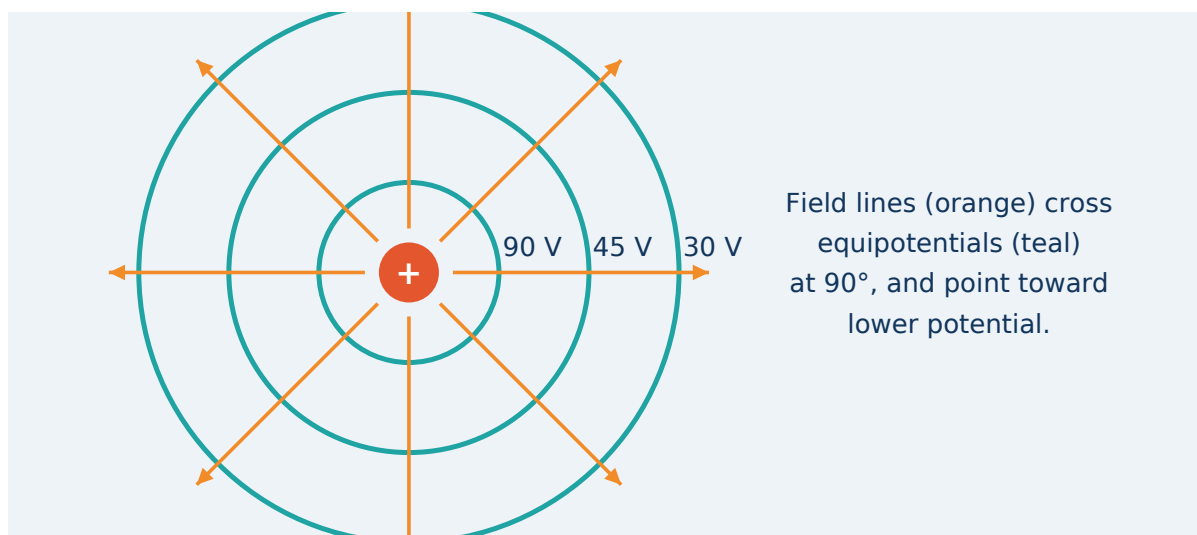
Data: a ring of radius $R = 0.30 \text{ m}$ carries $Q = 2.0 \text{ nC}$. **Find** V and E_x on the axis at $x = 0.40 \text{ m}$.

Method: $s = \sqrt{(0.30^2 + 0.40^2)} = 0.50 \text{ m}$, so $V = kQ/s = (8.99 \times 10^9)(2.0 \times 10^{-9})/0.50$. Then $E_x = -dV/dx = kQx/(x^2 + R^2)^{3/2}$.

Answer: $V = 36 \text{ V}$ and $E_x = 57.5 \text{ V/m}$, using $E_x = (17.98 \times 0.40)/0.125$.

What are equipotentials and why are conductors special?

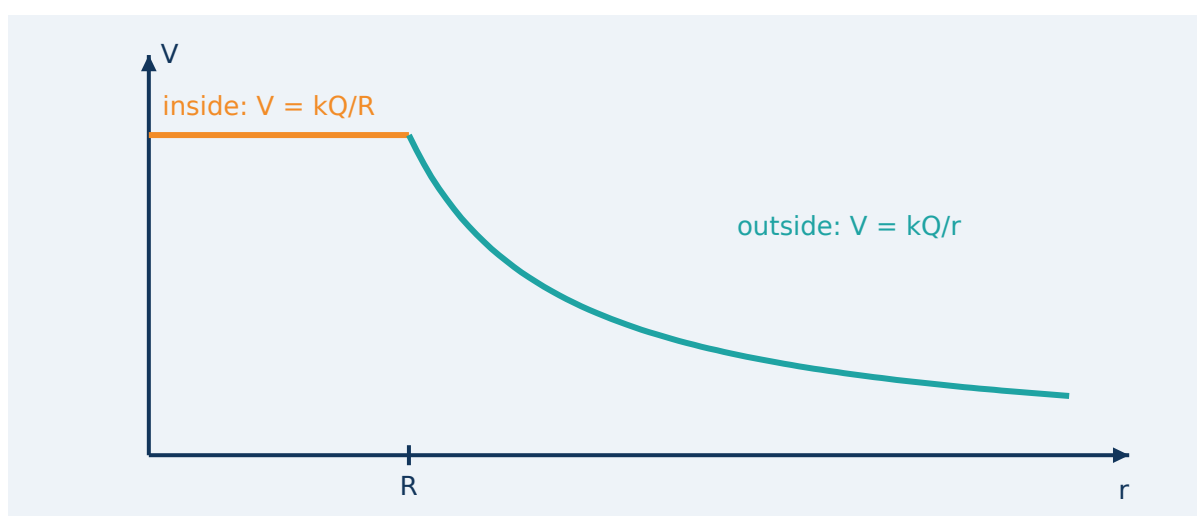
An equipotential is a surface on which V is constant. No work is done moving a charge along it, so the field must be perpendicular to it everywhere. Closely spaced equipotentials mean a strong field, because $|E| = |dV/dn|$ is the steepness of the potential.



Around a point charge the equipotentials are spheres; the field lines cross them at right angles.

Source	Potential V ($V = 0$ at infinity)
Point charge q	kq/r
Ring, on axis at x	$kQ/\sqrt{(x^2 + R^2)}$
Uniform field E , distance d along it	$V_0 - Ed$
Conducting sphere (R, Q), $r \geq R$	kQ/r
Conducting sphere, $r \leq R$	kQ/R (constant)
Infinite line, charge λ per metre	$-2k\lambda \ln r + \text{constant}$

In electrostatic equilibrium a conductor has $E = 0$ inside, so the whole conductor, surface included, is at one potential. Charge sits on the surface and gathers where the surface is most sharply curved, which is why points discharge first. Two conductors joined by a wire reach the same potential, not the same charge.



A charged conducting sphere is an equipotential inside; outside it behaves like a point charge.

Worked example

Data: a conducting sphere of radius 0.15 m is at 3000 V. **Find** its charge and the potential 0.45 m from its centre.

Method: $Q = VR/k = (3000)(0.15)/(8.99 \times 10^9)$ and $V(r) = V \cdot R/r$.

Answer: $Q = 50 \text{ nC}$ and $V = 1000 \text{ V}$ at 0.45 m .

Common mistake

Zero potential does not mean zero field. At the midpoint of two equal and opposite charges $V = 0$ yet E is large. The field depends on the slope of V , not its value.

Lina's tip

To get E from a V - x graph, read the slope, then flip its sign. A graph falling to the right means a field pointing right.

Key points to remember

- $V = U/q$ ($1 \text{ V} = 1 \text{ J/C}$); point charge $V = kq/r$, with $V = 0$ at infinity.
- $\Delta V = -\int E \cdot dl$ is path-independent; $E = -dV/dx$ gives the field from a slope.
- Potentials add as scalars: $V = k\int dq/s$ for continuous charge.
- Equipotentials are perpendicular to field lines; E points toward falling V .
- A conductor in equilibrium has $E = 0$ inside and is one equipotential.

Potential from fields and equipotentials: frequently asked questions

Is potential the same as potential energy?

No. Potential energy U belongs to a charge in a field, while potential $V = U/q$ belongs to a point in space. A $2 \text{ } \mu\text{C}$ charge at a 100 V point has $U = 2 \times 10^{-4} \text{ J}$.

Why does the field point toward lower potential?

Since $\Delta V = -\int E \cdot dl$, moving along the field gives a negative ΔV . A positive charge released from rest therefore accelerates toward lower potential, gaining kinetic energy as it loses potential energy.

Can V be zero where E is not zero?

Yes. $V = 0$ only tells you the value, while E depends on how fast V changes. At the midpoint of equal and opposite charges $V = 0$ but E is non-zero.

Why is the surface of a conductor an equipotential?

If two surface points had different V , a field would exist along the surface and push charges until it vanished. In equilibrium the tangential E is zero, so V is constant.

When do I use calculus for potential?

Use integration when the field varies with position or the charge is continuous. Use $\Delta V = -\int E \cdot dl$ for a known field and $V = k\int dq/s$ for a charge distribution, then differentiate to get E .