

Potential from fields and equipotentials

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These twenty problems train you to move between field, potential and energy using calculus: integrating E to get V , differentiating V to get E , and using conductors and equipotentials. They progress from one-step recall to multi-step derivations in the style of AP Physics C: Electricity and Magnetism.

The set suits students on the AP course, first-year university physics and A Level learners ready for calculus. Work with $k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 9

Problem 1 Units of potential (easy)

1. State the SI unit of electric potential in terms of joules and coulombs.
2. Calculate the work an external agent must do to carry a $3.0 \mu\text{C}$ charge through a potential difference of 200 V .

Problem 2 Point charge potential (easy)

1. Calculate the potential 0.30 m from a $+5.0 \text{ nC}$ point charge, taking $V = 0$ at infinity.
2. State the potential at 0.60 m from the same charge.

Problem 3 Uniform field gap (easy)

Aarav measures a uniform field of 1500 N/C between two parallel plates 0.020 m apart.

1. Calculate the potential difference between the plates.
2. State which plate is at the higher potential if the field points from plate A to plate B.

Problem 4 Electron accelerated (easy)

Chiamaka's school demonstration tube accelerates electrons from rest through 250 V .

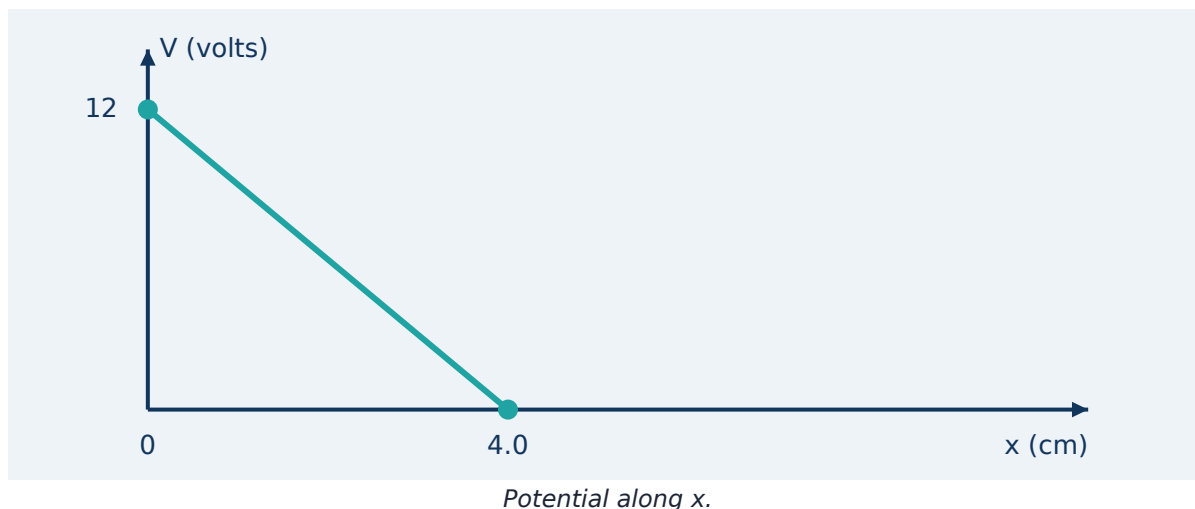
1. Calculate the kinetic energy gained, in joules.
2. Calculate the final speed ($m = 9.11 \times 10^{-31} \text{ kg}$, $e = 1.60 \times 10^{-19} \text{ C}$).

Problem 5 Inside a conductor (easy)

1. State the electric field inside a solid conductor in electrostatic equilibrium.
2. Explain why the whole conductor is at one potential.

Problem 6 Reading a V-x graph (easy)

The graph shows the potential along the x-axis in a region of space.



1. Calculate the magnitude of E in the region.
2. State the direction of E .
3. Calculate the work done by the field on a $+2.0 \mu\text{C}$ charge moved from $x = 0$ to $x = 4.0$ cm.

Problem 7 Two charges, one point (easy)

Charges of $+3.0 \text{ nC}$ and -1.0 nC are 0.40 m apart.

1. Calculate the potential at the midpoint.
2. Explain why you can add the two potentials without using vectors.

Problem 8 Field given by x^2 (medium)

Along the x-axis $E_x = (600 \text{ V/m}^3)x^2$, and $V(0) = 0$.

1. Integrate to find $V(x)$.
2. Calculate V at $x = 0.30 \text{ m}$.

Problem 9 Field from potential (medium)

A potential varies as $V(x) = 40x - 150x^2$ (V, with x in metres).

1. Derive $E_x(x)$.
2. Calculate E_x at $x = 0.10 \text{ m}$.
3. Find where $E_x = 0$.

Problem 10 Ring on its axis (medium)

A ring of radius 0.12 m carries a uniform charge of 4.0 nC .

1. Calculate the potential on the axis 0.16 m from the centre.
2. Calculate E_x at that point.

Problem 11 Assembling two charges (medium)

Wanjiru brings a $2.0\ \mu\text{C}$ charge from infinity to $0.50\ \text{m}$ from a fixed $3.0\ \mu\text{C}$ charge.

1. Calculate the work she must do.
2. Explain the sign of your answer.

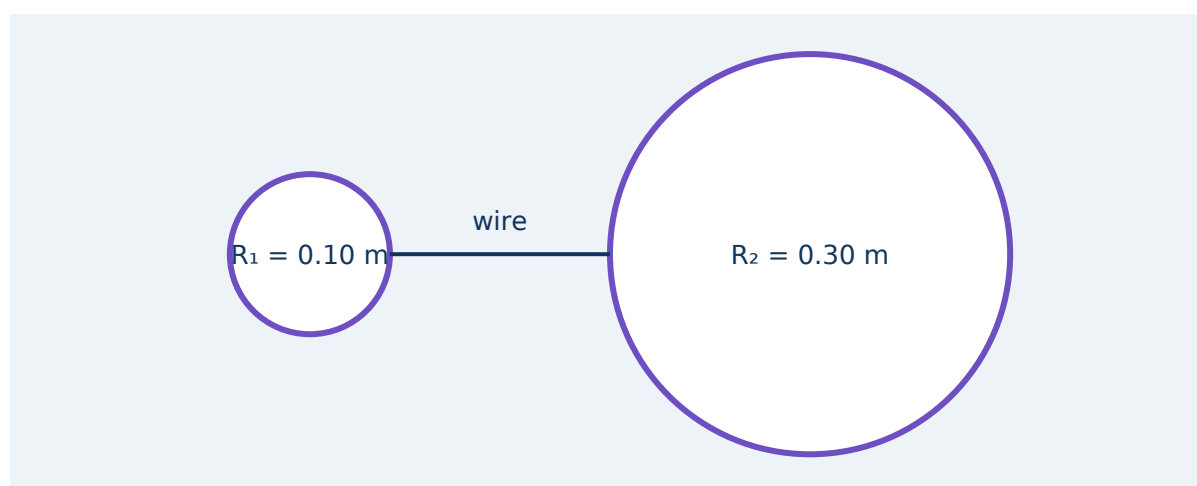
Problem 12 Charged sphere (medium)

A conducting sphere of radius $0.15\ \text{m}$ is held at $3000\ \text{V}$.

1. Calculate its charge.
2. Calculate the field at its surface.
3. Calculate the potential $0.45\ \text{m}$ from the centre.
4. State V and E inside.

Problem 13 Joined spheres (medium)

The two spheres shown are joined by a thin wire and share a total charge of $80\ \text{nC}$.

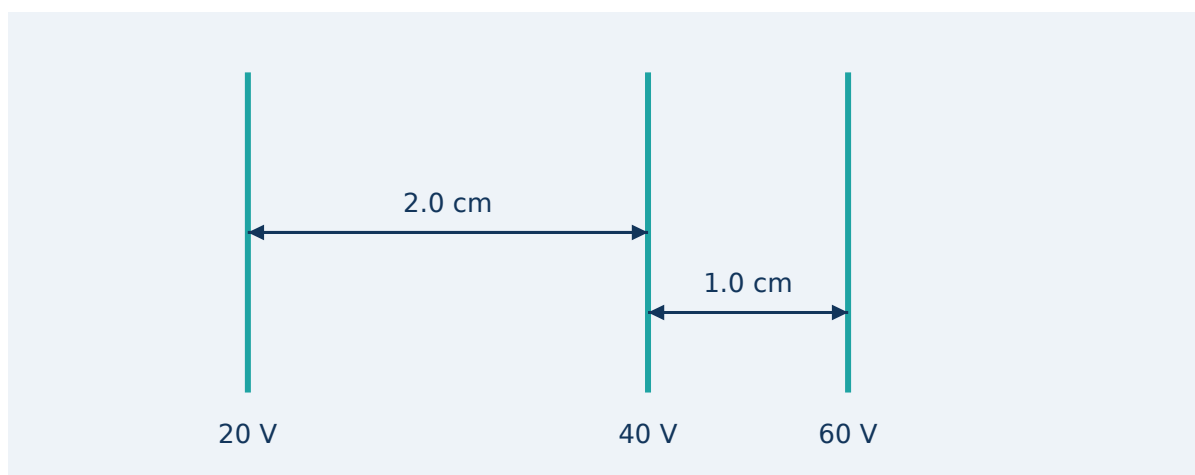


Two spheres joined by a wire.

1. Explain why they reach the same potential.
2. Calculate the charge on each sphere.
3. Calculate the common potential.
4. Compare the surface fields E_1 and E_2 .

Problem 14 Spacing of equipotentials (medium)

The diagram shows three equipotential lines in a flat region.



Equipotentials at 20 V, 40 V and 60 V.

1. Calculate the average field between 20 V and 40 V.
2. Calculate the average field between 40 V and 60 V.
3. Explain where the field is stronger.
4. State the direction of the field.

Problem 15 Potential near a long wire (medium)

A long straight wire carries a uniform charge of 3.0 nC per metre, so $E = 2k\lambda/r$.

1. Derive the potential difference $V(0.10 \text{ m}) - V(0.40 \text{ m})$.
2. Calculate it and state which point is at higher potential.

Problem 16 Charged rod on its axis (hard)

A thin rod of length 0.50 m carries 6.0 nC uniformly. Point P lies on the rod's line, 0.10 m beyond one end.

1. Show that $V = k(Q/L) \ln[(L + d)/d]$ for distance d from the near end.
2. Calculate V at P.

Problem 17 Charged disc (hard)

A disc of radius 0.20 m carries uniform surface charge 5.0 nC/m². Its potential on the axis is $V = (\sigma/2\epsilon_0)(\sqrt{x^2 + R^2} - x)$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$.

1. Calculate V at $x = 0.15 \text{ m}$.
2. Differentiate to find E_x and calculate it at $x = 0.15 \text{ m}$.
3. Show that V tends to kQ/x for large x .

Problem 18 Uniform charged ball (hard)

Kofi models a cloud as a uniformly charged ball of radius 0.050 m holding 10 nC, with $E = kQr/R^3$ inside.

1. Integrate to find $V(r)$ inside, matching kQ/R at the surface.
2. Calculate V at the centre.

3. Calculate V at $r = R/2$.

Problem 19 Particle released near a charge (hard)

A 5.0 g bead carrying $+2.0 \mu\text{C}$ is released from rest 0.10 m from a fixed $+4.0 \mu\text{C}$ charge, on a frictionless insulating track.

1. Calculate its speed when it reaches $r = 0.50$ m.
2. Calculate its speed as $r \rightarrow \infty$.
3. Explain why the speed never exceeds the second value.

Problem 20 Spot the error (hard)

Mei says: 'Inside a hollow charged conducting shell $E = 0$, so $V = 0$ there too.' The shell has radius 0.25 m and charge -30 nC .

1. Identify and correct her mistake.
2. Calculate the potential inside the shell.
3. Calculate the potential 0.50 m from the centre.