

Potential from fields and equipotentials

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Full methods follow for every problem: formula, substitution and the result with its unit in bold. Check each line against your own working, and look for the reasoning about signs and directions, which is where most marks are lost.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 9

Problem 1 Units of potential

1. $V = J/C$, so **1 V = 1 J/C**.
2. $W = q\Delta V = (3.0 \times 10^{-6})(200) = \mathbf{6.0 \times 10^{-4} J}$.

Problem 2 Point charge potential

1. $V = kq/r = (8.99 \times 10^9)(5.0 \times 10^{-9})/0.30 = \mathbf{150 V}$ (149.8 V).
2. Doubling r halves V : **75 V**.

Problem 3 Uniform field gap

1. $\Delta V = Ed = (1500)(0.020) = \mathbf{30 V}$.
2. The field points toward lower potential, so plate A is higher: **plate A**.

Problem 4 Electron accelerated

1. $KE = e\Delta V = (1.60 \times 10^{-19})(250) = \mathbf{4.0 \times 10^{-17} J}$.
2. $v = \sqrt{(2KE/m)} = \sqrt{(8.0 \times 10^{-17}/9.11 \times 10^{-31})} = \mathbf{9.4 \times 10^6 m/s}$.

Problem 5 Inside a conductor

1. **$E = 0$** .
2. With $E = 0$ everywhere inside, $\Delta V = -\int E \cdot dl = 0$ between any two points, and the surface is part of the same body, so the conductor is **one equipotential**.

Problem 6 Reading a V-x graph

1. $|E| = |\Delta V/\Delta x| = 12/0.040 = \mathbf{300 V/m}$.
2. V falls as x increases, so E points in the **+x direction**.
3. $W = q(V_i - V_f) = (2.0 \times 10^{-6})(12 - 0) = \mathbf{2.4 \times 10^{-5} J}$.

Problem 7 Two charges, one point

1. $V = k(q_1 + q_2)/r = (8.99 \times 10^9)(3.0 - 1.0)(10^{-9})/0.20 = 17.98/0.20 = \mathbf{90 \text{ V}}$.
2. Potential is a scalar, so contributions **add algebraically**, with sign.

Problem 8 Field given by x^2

1. $V(x) = -\int \text{from } 0 \text{ to } x \text{ of } 600x'^2 \, dx' = \mathbf{-200x^3}$.
2. $V(0.30) = -200(0.027) = \mathbf{-5.4 \text{ V}}$.

Problem 9 Field from potential

1. $E_x = -dV/dx = -(40 - 300x) = \mathbf{300x - 40 \text{ (V/m)}}$.
2. $E_x(0.10) = 30 - 40 = \mathbf{-10 \text{ V/m}}$.
3. $300x = 40$ gives $x = \mathbf{0.133 \text{ m}}$.

Problem 10 Ring on its axis

1. $s = \sqrt{(0.12^2 + 0.16^2)} = 0.20 \text{ m}$, $V = kQ/s = (8.99 \times 10^9)(4.0 \times 10^{-9})/0.20 = \mathbf{180 \text{ V}}$.
2. $E_x = kQx/s^3 = (35.96)(0.16)/0.0080 = \mathbf{7.2 \times 10^2 \text{ V/m}}$.

Problem 11 Assembling two charges

1. $W = kq_1q_2/r = (8.99 \times 10^9)(2.0 \times 10^{-6})(3.0 \times 10^{-6})/0.50 = \mathbf{0.11 \text{ J}}$.
2. W is positive because the charges repel, so Wanjiru does positive work and the pair stores that energy.

Problem 12 Charged sphere

1. $Q = VR/k = (3000)(0.15)/(8.99 \times 10^9) = \mathbf{5.0 \times 10^{-8} \text{ C}}$.
2. $E = V/R = 3000/0.15 = \mathbf{2.0 \times 10^4 \text{ V/m}}$.
3. $V = 3000(0.15/0.45) = \mathbf{1000 \text{ V}}$.
4. Inside, $V = \mathbf{3000 \text{ V}}$ and $E = 0$.

Problem 13 Joined spheres

1. The wire is a conductor, so charge flows until there is no field along it, which means equal potentials.
2. $kQ_1/R_1 = kQ_2/R_2$ gives $Q \propto R$, so $Q_1 : Q_2 = 1 : 3$, hence $\mathbf{Q_1 = 20 \text{ nC}}$ and $\mathbf{Q_2 = 60 \text{ nC}}$.
3. $V = kQ_1/R_1 = (8.99 \times 10^9)(2.0 \times 10^{-8})/0.10 = \mathbf{1.8 \times 10^3 \text{ V}}$.
4. $E = V/R$, so $E_1/E_2 = R_2/R_1 = \mathbf{3}$: the small sphere has the stronger surface field.

Problem 14 Spacing of equipotentials

1. $E = 20/0.020 = \mathbf{1.0 \times 10^3 \text{ V/m}}$.
2. $E = 20/0.010 = \mathbf{2.0 \times 10^3 \text{ V/m}}$.

3. The field is stronger where the lines are closer, between 40 V and 60 V.
4. The field points toward lower potential, to the **left**, perpendicular to the lines.

Problem 15 Potential near a long wire

1. $V(0.10) - V(0.40) = -\int \text{from } 0.40 \text{ to } 0.10 \text{ of } (2k\lambda/r) dr = 2k\lambda \ln(0.40/0.10)$.
2. $2k\lambda = 2(8.99 \times 10^9)(3.0 \times 10^{-9}) = 53.94 \text{ V}$, times $\ln 4 = 1.386$ gives **75 V**. The point 0.10 m from the wire is at the higher potential.

Problem 16 Charged rod on its axis

1. Piece $dq = (Q/L)dx$ at distance x from P: $V = k(Q/L)\int \text{from } d \text{ to } d+L \text{ of } dx/x = k(Q/L) \ln[(L + d)/d]$.
2. $V = (8.99 \times 10^9)(1.2 \times 10^{-8}) \ln(0.60/0.10) = 107.9 \times 1.792 = \mathbf{1.9 \times 10^2 \text{ V}}$ (193 V).

Problem 17 Charged disc

1. $\sigma/2\epsilon_0 = 5.0 \times 10^{-9}/(1.77 \times 10^{-11}) = 282.5 \text{ V/m}$, and $\sqrt{(0.15^2 + 0.20^2)} - 0.15 = 0.10 \text{ m}$, so $V = \mathbf{28 \text{ V}}$.
2. $E_x = -dV/dx = (\sigma/2\epsilon_0)(1 - x/\sqrt{x^2 + R^2}) = 282.5(1 - 0.60) = \mathbf{1.1 \times 10^2 \text{ V/m}}$.
3. For $x \gg R$, $\sqrt{x^2 + R^2} \approx x + R^2/2x$, so $V \approx (\sigma/2\epsilon_0)(R^2/2x) = \sigma\pi R^2/(4\pi\epsilon_0 x) = \mathbf{kQ/x}$.

Problem 18 Uniform charged ball

1. $V(r) = V(R) + \int \text{from } r \text{ to } R \text{ of } kQr'/R^3 dr' = kQ/R + kQ(R^2 - r^2)/(2R^3) = \mathbf{kQ(3R^2 - r^2)/(2R^3)}$.
2. Centre: $V = 3kQ/2R = 1.5(89.9)/0.050 = \mathbf{2.7 \times 10^3 \text{ V}}$.
3. At $R/2$: $V = 1.375 kQ/R = 1.375(1798) = \mathbf{2.5 \times 10^3 \text{ V}}$.

Problem 19 Particle released near a charge

1. $U = kQq/r$: $U(0.10) = 0.7192 \text{ J}$ and $U(0.50) = 0.1438 \text{ J}$, so $KE = 0.5754 \text{ J}$ and $v = \sqrt{(2KE/m)} = \mathbf{15 \text{ m/s}}$.
2. All 0.7192 J becomes kinetic: $v = \sqrt{(2 \times 0.7192/0.0050)} = \mathbf{17 \text{ m/s}}$.
3. The potential energy cannot fall below zero, so no more energy is available to convert.

Problem 20 Spot the error

1. Zero field inside only means V is constant there, not zero. Inside the shell V equals its surface value.
2. $V = kQ/R = (8.99 \times 10^9)(-30 \times 10^{-9})/0.25 = \mathbf{-1.1 \times 10^3 \text{ V}}$.
3. $V = kQ/r = -269.7/0.50 = \mathbf{-5.4 \times 10^2 \text{ V}}$.