

Faraday's law, inductors and LR circuits

AP Physics C (Mechanics and E&M, calculus-based) · Lesson

Quick answer: Faraday's law states that a changing magnetic flux through a coil induces an emf equal to $-N d\Phi/dt$, with its direction set by Lenz's law. Self-inductance L links this emf to changing current: $\varepsilon = -L dI/dt$. In an LR circuit current changes exponentially with time constant $\tau = L/R$.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 13

What does Faraday's law say?

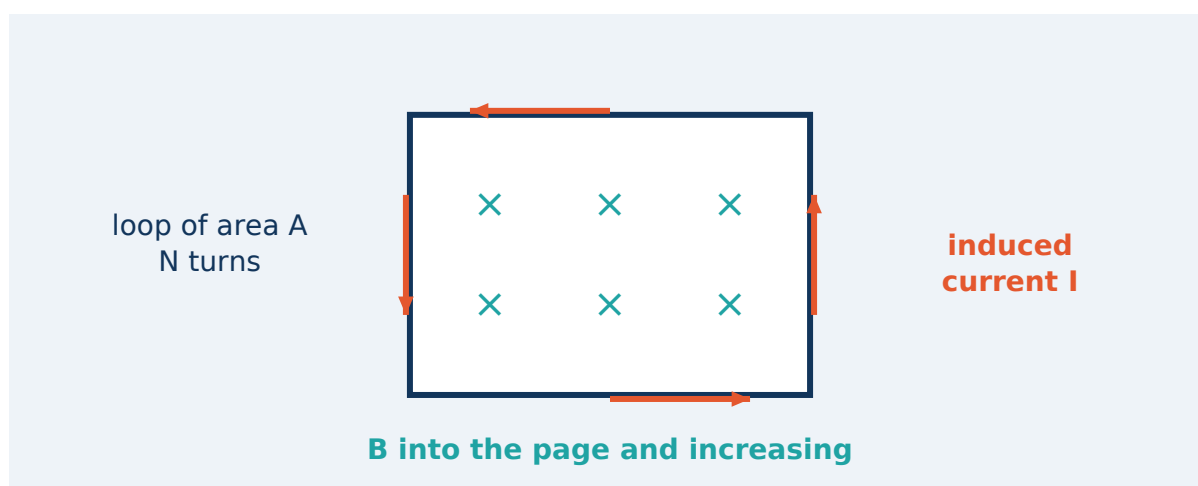
Definition

Magnetic flux through a surface is $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$, measured in webers ($1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$). For a uniform field it becomes $\Phi_B = BA \cos \theta$, where θ is the angle between the field and the normal to the surface.

When the flux linked with a coil changes, an emf appears in the coil. The size of the effect depends only on how fast the flux changes and on the number of turns, not on what makes the flux change.

Key law

Faraday's law: $\varepsilon = -N d\Phi_B/dt$. The minus sign is Lenz's law: the induced current flows in the direction whose own magnetic field opposes the change in flux.



Lenz's law: the loop opposes the growing inward flux, so the induced current is anticlockwise and its own field points out of the page.

The flux can change because B changes, because the area changes, or because the angle changes. A loop of area A in a field $B(t) = B_0 \sin \omega t$ has flux $B_0 A \sin \omega t$, so the emf is a cosine: $\varepsilon = -NB_0 A \omega \cos \omega t$.

How is motional emf found?

A straight conductor of length ℓ moving at speed v perpendicular to a field B sweeps out area at the rate ℓv . Faraday's law then gives a motional emf, and the magnetic force on the charges in the rod does the pushing.

Formula

$\varepsilon = B\ell v$, with ε in volts (V), B in tesla (T), ℓ in metres (m) and v in metres per second (m/s).

Worked example

A coil has $N = 200$ turns and area $4.0 \times 10^{-4} \text{ m}^2$. A field perpendicular to the coil falls uniformly from 0.50 T to zero in 0.20 s. The emf magnitude is $N A \Delta B / \Delta t = 200 \times (4.0 \times 10^{-4}) \times 0.50 / 0.20 = \mathbf{0.20 \text{ V}}$.

What is inductance?

A coil that carries a changing current produces a changing flux through itself, so it induces an emf in itself. This self-induction is described by the inductance L , measured in henries ($1 \text{ H} = 1 \text{ V}\cdot\text{s/A}$).

Formula

$L = N\Phi_B / I$ and $\varepsilon_L = -L \, dI/dt$. Energy stored: $U = \frac{1}{2}LI^2$, in joules. An ideal long solenoid has $L = \mu_0 N^2 A / \ell$, where N is the number of turns, A the cross-sectional area and ℓ the length.

Worked example

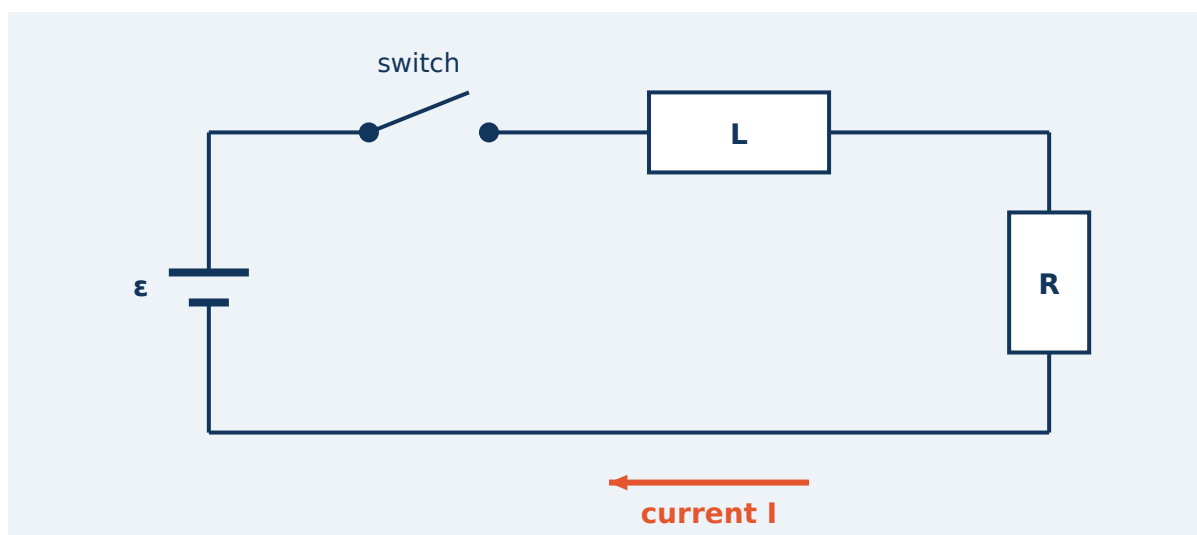
A solenoid has $N = 800$, $A = 2.0 \times 10^{-4} \text{ m}^2$ and $\ell = 0.40 \text{ m}$. Then $L = (4\pi \times 10^{-7})(800)^2(2.0 \times 10^{-4})/0.40 = 4.0 \times 10^{-4} \text{ H} = \mathbf{0.40 \text{ mH}}$.

How does current grow in an LR circuit?

When a switch connects a battery of emf ε to an inductor and resistor in series, Kirchhoff's loop rule gives $\varepsilon - IR - L \, dI/dt = 0$. Solving it gives an exponential approach to the final current. At the first instant the inductor carries all the voltage, so $dI/dt = \varepsilon/L$.

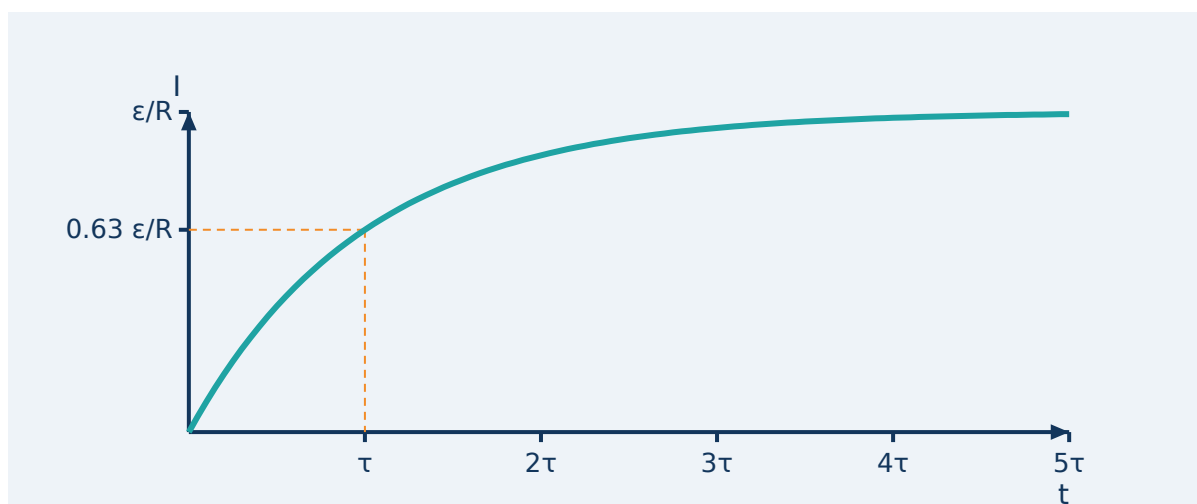
Formula

Growth: $I(t) = (\varepsilon/R)(1 - e^{-t/\tau})$. Decay: $I(t) = I_0 e^{-t/\tau}$. The time constant is $\tau = L/R$, in seconds.



Series LR circuit: the inductor makes the current rise gradually after the switch is closed.

Time	Fraction of final current	Fraction of initial current in decay
0	0	1
τ	0.632	0.368
2τ	0.865	0.135
3τ	0.950	0.050
5τ	0.993	0.007



Current growth: I reaches 63 % of its final value after one time constant $\tau = L/R$.

Worked example

$L = 0.50 \text{ H}$, $R = 25 \Omega$, $\varepsilon = 10 \text{ V}$. Then $\tau = 0.50/25 = 0.020 \text{ s}$ and the final current is $10/25 = 0.40 \text{ A}$. At $t = 0.040 \text{ s} = 2\tau$, $I = 0.40 (1 - e^{-2}) = \mathbf{0.35 \text{ A}}$.

Common mistake

Students say an inductor opposes current. It opposes the change in current. At steady state it has no emf across it and behaves like a plain wire.

What happens in an LC circuit?

A charged capacitor connected to an inductor exchanges energy between its electric field and the inductor's magnetic field. With no resistance the total $\frac{1}{2}CV^2 + \frac{1}{2}LI^2$ stays constant and the charge oscillates with angular frequency $\omega = 1/\sqrt{LC}$, so the period is $T = 2\pi\sqrt{LC}$. Resistance adds damping.

Lina's tip

To find the sign in Lenz's law, ask what the loop wants: it fights the change. If flux into the page grows, the loop produces flux out of the page.

Key points to remember

- Faraday's law: $\varepsilon = -N d\Phi/dt$, flux in $\text{Wb} = \text{T}\cdot\text{m}^2$
- Motional emf of a rod: $\varepsilon = Blv$
- Inductance: $L = N\Phi/I$, solenoid $L = \mu_0 N^2 A/\ell$, energy $U = \frac{1}{2}LI^2$
- LR circuit: $\tau = L/R$, $I = (\varepsilon/R)(1 - e^{-(t/\tau)})$
- LC circuit: $\omega = 1/\sqrt{LC}$, energy swaps between C and L

Faraday's law, inductors and LR circuits: frequently asked questions

What is Faraday's law of induction?

The emf induced in a coil equals the negative rate of change of magnetic flux linkage: $\varepsilon = -N d\Phi/dt$. It applies whether the flux changes because of field, area or angle.

What does the minus sign in Faraday's law mean?

It is Lenz's law. The induced current produces a field that opposes the change in flux, which is how energy is conserved.

What is the time constant of an LR circuit?

It is $\tau = L/R$. After one time constant the current has covered 63 % of its change towards the final value.

How much energy does an inductor store?

An inductor carrying current I stores $U = \frac{1}{2}LI^2$ in its magnetic field. The energy is released when the current falls.

How do you find the frequency of an LC circuit?

Use $\omega = 1/\sqrt{LC}$ in rad/s, then $f = \omega/2\pi$. A larger inductance or capacitance gives a lower frequency.