

Faraday's law, inductors and LR circuits

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the formula, the substitution and the result in bold. Check your method as well as your final value, because free-response marks reward the reasoning.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 13

Problem 1 Flux through a loop

1. $\Phi = BA = 0.30 \times 0.020 = \mathbf{6.0 \times 10^{-3} \text{ Wb}}$.
2. $\Phi = BA \cos 60^\circ = 6.0 \times 10^{-3} \times 0.50 = \mathbf{3.0 \times 10^{-3} \text{ Wb}}$.
3. $1 \text{ Wb} = \mathbf{1 \text{ T} \cdot \text{m}^2}$.

Problem 2 Emf from a changing flux

1. $\Delta\Phi = 0.5 - 2.0 = -1.5 \text{ mWb} = -1.5 \times 10^{-3} \text{ Wb}$.
2. $|\mathcal{E}| = N|\Delta\Phi|/\Delta t = 50 \times 1.5 \times 10^{-3} / 0.10 = \mathbf{0.75 \text{ V}}$.
3. $1 \text{ Wb} = 1 \text{ V} \cdot \text{s}$, so $1 \text{ Wb/s} = \mathbf{1 \text{ V}}$.

Problem 3 Direction of the induced current

1. The induced field opposes the growing inward field, so it points **out of the page**.
2. An out-of-page field inside the loop needs an **anticlockwise** current (right-hand rule).
3. If the field decreased, the loop would try to keep the inward flux, so the induced field would point **into the page** and the current would be clockwise.

Problem 4 Motional emf of a rod

1. $\mathcal{E} = Blv = 0.25 \times 0.30 \times 4.0 = \mathbf{0.30 \text{ V}}$.
2. $I = \mathcal{E}/R = 0.30/0.50 = \mathbf{0.60 \text{ A}}$.

Problem 5 Inductance of a coil

1. $N\Phi = 400 \times 2.5 \times 10^{-5} = 1.0 \times 10^{-2} \text{ Wb}$.
2. $L = N\Phi/I = 1.0 \times 10^{-2}/0.50 = 0.020 \text{ H} = \mathbf{20 \text{ mH}}$.

Problem 6 Self-induced emf

1. $dI/dt = 2.0/0.050 = 40 \text{ A/s}$.
2. $|\mathcal{E}| = L dI/dt = 0.040 \times 40 = \mathbf{1.6 \text{ V}}$.

Problem 7 Time constant

1. $\tau = L/R = 0.30/60 = 5.0 \times 10^{-3} \text{ s} = \mathbf{5.0 \text{ ms}}$.
2. $5\tau = \mathbf{25 \text{ ms}}$ (the current is then about 99.3 % of its final value).

Problem 8 Energy in an inductor

1. $U = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.25 \times 3.0^2 = 1.125 \text{ J} \approx \mathbf{1.1 \text{ J}}$.
2. U depends on I^2 , so doubling I gives **a factor of 4** (4.5 J).

Problem 9 Inductance of a solenoid

1. $A = \pi r^2 = \pi(0.015)^2 = 7.07 \times 10^{-4} \text{ m}^2$.
2. $L = \mu_0 N^2 A / \ell = (4\pi \times 10^{-7})(1200)^2(7.07 \times 10^{-4})/0.60 = 2.13 \times 10^{-3} \text{ H} \approx \mathbf{2.1 \text{ mH}}$.
3. $L \propto N^2$, so doubling N multiplies L by **4**.

Problem 10 Alternating emf from a changing field

1. $\Phi = BA = 0.40 \times 0.010 \sin(100\pi t) = 4.0 \times 10^{-3} \sin(100\pi t) \text{ Wb}$.
2. $\varepsilon = -N d\Phi/dt = -100 \times 4.0 \times 10^{-3} \times 100\pi \cos(100\pi t) = -40\pi \cos(100\pi t) \text{ V}$.
3. $\varepsilon_{\max} = 40\pi = 125.7 \text{ V} \approx \mathbf{1.3 \times 10^2 \text{ V}}$.

Problem 11 Reading a current graph

1. The curve levels off at **0.60 A**.
2. 63 % of 0.60 A is 0.38 A, reached at 12 ms, so $\tau = \mathbf{12 \text{ ms}}$.
3. $R = \varepsilon/I = 12/0.60 = 20 \Omega$. $L = \tau R = 0.012 \times 20 = 0.24 \text{ H}$, so **L = 0.24 H**.

Problem 12 Growth of current

1. $\tau = L/R = 0.80/40 = \mathbf{0.020 \text{ s}}$.
2. $I_{\infty} = 24/40 = 0.60 \text{ A}$. At 40 ms = 2τ , $I = 0.60 (1 - e^{-2}) = 0.60 \times 0.8647 = \mathbf{0.52 \text{ A}}$.

Problem 13 Decay of current

1. $I_0 = \varepsilon/R = 24/40 = \mathbf{0.60 \text{ A}}$.
2. 30 ms = 1.5τ , so $I = 0.60 e^{-1.5} = 0.60 \times 0.2231 = \mathbf{0.13 \text{ A}}$.

Problem 14 Initial rate of change

1. The current is zero, so $V_R = \mathbf{0}$.
2. The inductor carries the whole 24 V: $dl/dt = \varepsilon/L = 24/0.80 = \mathbf{30 \text{ A/s}}$.
3. At steady state $dl/dt = 0$, so the inductor has no emf and only R limits the current: $I = \varepsilon/R$, **independent of L**.

Problem 15 Frequency of an LC circuit

- $\omega = 1/\sqrt{LC} = 1/\sqrt{(2.0 \times 10^{-3} \times 50 \times 10^{-6})} = 1/\sqrt{(1.0 \times 10^{-7})} = 3.16 \times 10^3 \text{ rad/s} \approx \mathbf{3.2 \times 10^3 \text{ rad/s}}$.
- $f = \omega/2\pi = 503 \text{ Hz} \approx \mathbf{5.0 \times 10^2 \text{ Hz}}$.
- $\omega \propto 1/\sqrt{C}$, so multiplying C by 4 **halves** the frequency.

Problem 16 Terminal speed of a falling rod

- The emf is $B\ell v$, the current is $B\ell v/R$, so $F = BIl = B^2\ell^2 v/R$ (upwards).
- Set $B^2\ell^2 v/R = mg$: $v = mgR/(B^2\ell^2) = (0.050 \times 9.81 \times 0.20)/(0.25 \times 0.16) = 0.0981/0.040 = 2.45 \text{ m/s} \approx \mathbf{2.5 \text{ m/s}}$.
- At terminal speed the kinetic energy is constant, so all the gravitational energy lost heats the resistor ($mgv = I^2 R$).

Problem 17 Decaying field through a loop

- $A = \pi(0.050)^2 = \mathbf{7.85 \times 10^{-3} \text{ m}^2}$.
- $dB/dt = -(0.80/0.40) e^{-t/0.40}$, so at $t = 0$ it is -2.0 T/s . $|\epsilon| = A|dB/dt| = 7.85 \times 10^{-3} \times 2.0 = 1.57 \times 10^{-2} \text{ V} \approx \mathbf{16 \text{ mV}}$.
- At $t = 0.40 \text{ s}$ multiply by $e^{-1} = 0.368$: $|\epsilon| = 15.7 \times 0.368 = 5.8 \text{ mV}$, so $\mathbf{5.8 \text{ mV}}$.

Problem 18 Energy dissipated while the current decays

- $U_0 = \frac{1}{2}LI_0^2 = \frac{1}{2} \times 0.80 \times 0.60^2 = \mathbf{0.144 \text{ J}}$.
- $U \propto I^2$, so $U(\tau) = U_0 e^{-2} = 0.144 \times 0.1353 = 0.0195 \text{ J} \approx \mathbf{0.019 \text{ J}}$.
- Dissipated = $0.144 - 0.0195 = 0.1245 \text{ J} \approx \mathbf{0.12 \text{ J}}$.

Problem 19 Charge and current in an LC circuit

- $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 1.0 \times 10^{-4} \times 12^2 = \mathbf{7.2 \times 10^{-3} \text{ J}}$.
- Energy conservation: $\frac{1}{2}LI_{\text{max}}^2 = \frac{1}{2}CV^2$, so $I_{\text{max}} = V\sqrt{C/L} = 12 \times \sqrt{(1.0 \times 10^{-4}/0.025)} = 12 \times 0.0632 = \mathbf{0.76 \text{ A}}$.
- $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(0.025 \times 1.0 \times 10^{-4})} = 2\pi \times 1.58 \times 10^{-3} = \mathbf{9.9 \text{ ms}}$.
- At $I = I_{\text{max}}/2$ the inductor holds $\frac{1}{4}$ of the energy, so the capacitor holds $\frac{3}{4}$: $q^2/2C = \frac{3}{4}Q^2/2C$, $q = (\sqrt{3}/2)Q$. With $Q = CV = 1.2 \times 10^{-3} \text{ C}$, $q = 0.866 \times 1.2 \times 10^{-3} = 1.04 \times 10^{-3} \text{ C} \approx \mathbf{1.0 \times 10^{-3} \text{ C}}$.

Problem 20 Timing in an LR circuit

- $I = I_{\infty}(1 - e^{-t/\tau}) = 0.9 I_{\infty}$ gives $t = \tau \ln 10$. With $\tau = 0.50/25 = 0.020 \text{ s}$, $t = 0.020 \times 2.303 = 0.046 \text{ s} = \mathbf{46 \text{ ms}}$.
- $U \propto I^2$, so half the final energy means $I = I_{\infty}/\sqrt{2} = 0.7071 I_{\infty}$. Then $e^{-t/\tau} = 0.2929$ and $t = \tau \ln(1/0.2929) = 0.020 \times 1.228 = 0.0246 \text{ s} \approx \mathbf{25 \text{ ms}}$.
- Half the energy needs only 71 % of the final current, and the current rises fastest at the

start, so that value is reached sooner than 90 %.