

# Heat, work and engines on PV diagrams

## AP Physics 2 (algebra-based) · Lesson

**Quick answer:** The first law of thermodynamics states  $\Delta U = Q + W$ : the internal energy of a gas changes by the heat added plus the work done on it. On a PV diagram the area under a process is the work. A heat engine has efficiency  $e = W/Q_H$ , limited by the Carnot value  $1 - T_C/T_H$ .

**Level:** AP Physics 2 (algebra-based) · **Curriculum:** AP P2 Unit 9

## What does the first law of thermodynamics say?

The first law is conservation of energy for a thermal system. The internal energy of a gas changes only when heat flows across its boundary or when work is done on it or by it. In AP Physics 2,  $W$  is the work done *on* the gas, so a compressed gas receives positive work and an expanding gas, which pushes on its surroundings, has negative  $W$ .

### Definition

**Internal energy**  $U$  is the total microscopic energy of the particles. For an ideal monatomic gas it is only translational kinetic energy,  $U = (3/2)nRT$ , so it depends on the temperature alone.

### Key law

$\Delta U = Q + W$ .  $Q > 0$  when heat flows into the gas;  $W > 0$  when the surroundings do work on the gas.

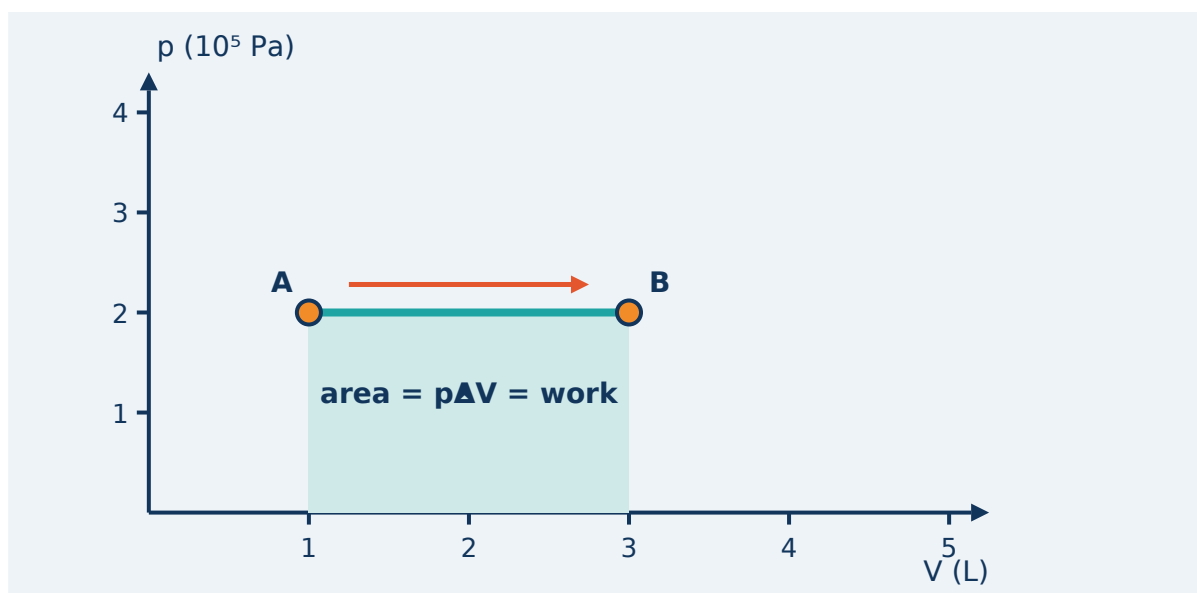
### Formula

$W = -p\Delta V$  (constant pressure)     $\Delta U = (3/2)nR\Delta T$  (monatomic ideal gas)

$W$  is the work done on the gas (J),  $p$  is the pressure (Pa),  $\Delta V$  is the change of volume ( $\text{m}^3$ ),  $n$  is the amount of gas (mol),  $R = 8.31 \text{ J}/(\text{mol}\cdot\text{K})$  and  $\Delta T$  is the temperature change (K).

## How do you find work from a PV diagram?

On a graph of pressure against volume, the area under the process curve equals the magnitude of the work. Moving to the right means the gas expands and does work on the surroundings, so  $W$  is negative. Moving to the left means compression and  $W$  is positive. For a closed cycle, the enclosed area is the net work, and a clockwise loop means the gas does net work on its surroundings.



The shaded area under the process line equals the magnitude of the work.

### Worked example

A gas expands at constant pressure  $2.0 \times 10^5$  Pa from  $1.0 \times 10^{-3}$  m<sup>3</sup> to  $3.0 \times 10^{-3}$  m<sup>3</sup> while absorbing 900 J of heat. Find  $W$  and  $\Delta U$ .

$$W = -p\Delta V = -2.0 \times 10^5 \times 2.0 \times 10^{-3} = \mathbf{-400 \text{ J}}. \Delta U = Q + W = 900 - 400 = \mathbf{+500 \text{ J}}.$$

## What are the four basic gas processes?

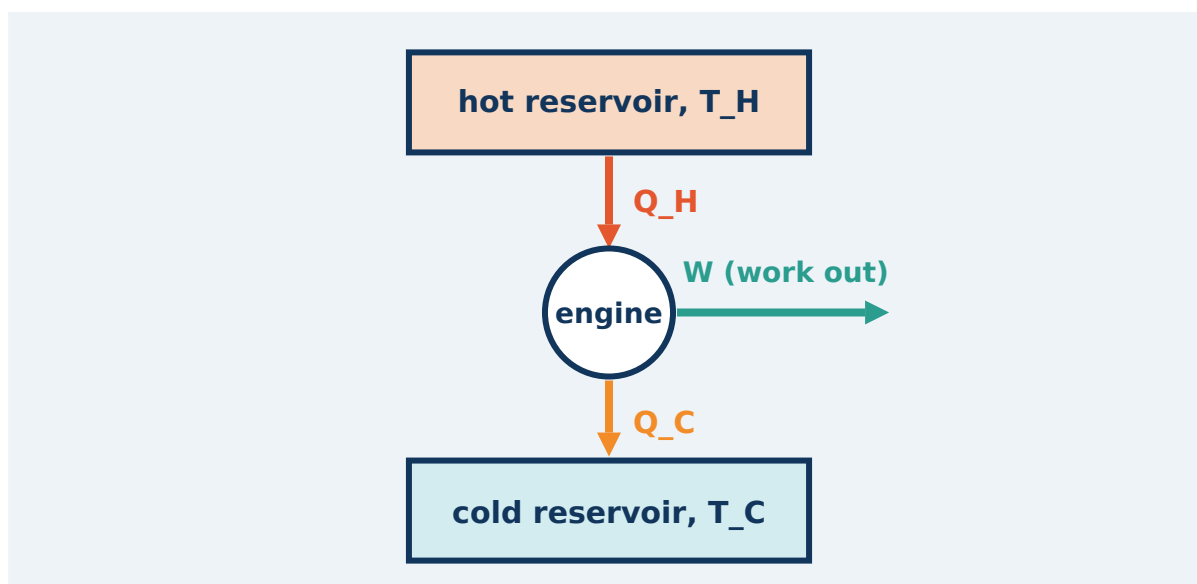
| Process    | What is fixed | Work $W$     | Heat $Q$       | $\Delta U$        |
|------------|---------------|--------------|----------------|-------------------|
| Isochoric  | volume        | 0            | $Q = \Delta U$ | $(3/2)nR\Delta T$ |
| Isobaric   | pressure      | $-p\Delta V$ | $\Delta U - W$ | $(3/2)nR\Delta T$ |
| Isothermal | temperature   | $-Q$         | $Q = -W$       | 0                 |
| Adiabatic  | no heat flow  | $\Delta U$   | 0              | $W$               |

### Common mistake

Do not mix sign conventions. If  $W$  means work done *on* the gas, then  $\Delta U = Q + W$  and an expansion has negative  $W$ . Some textbooks use work done *by* the gas with  $\Delta U = Q - W$ . Pick one and say which.

## How does a heat engine turn heat into work?

A heat engine takes heat  $Q_H$  from a hot reservoir, converts part of it to work and rejects the rest,  $Q_C$ , to a cold reservoir. After one full cycle the gas returns to its starting state, so  $\Delta U = 0$  and the net work equals  $Q_H - Q_C$ .



A heat engine:  $W = Q_H - Q_C$ .

### Formula

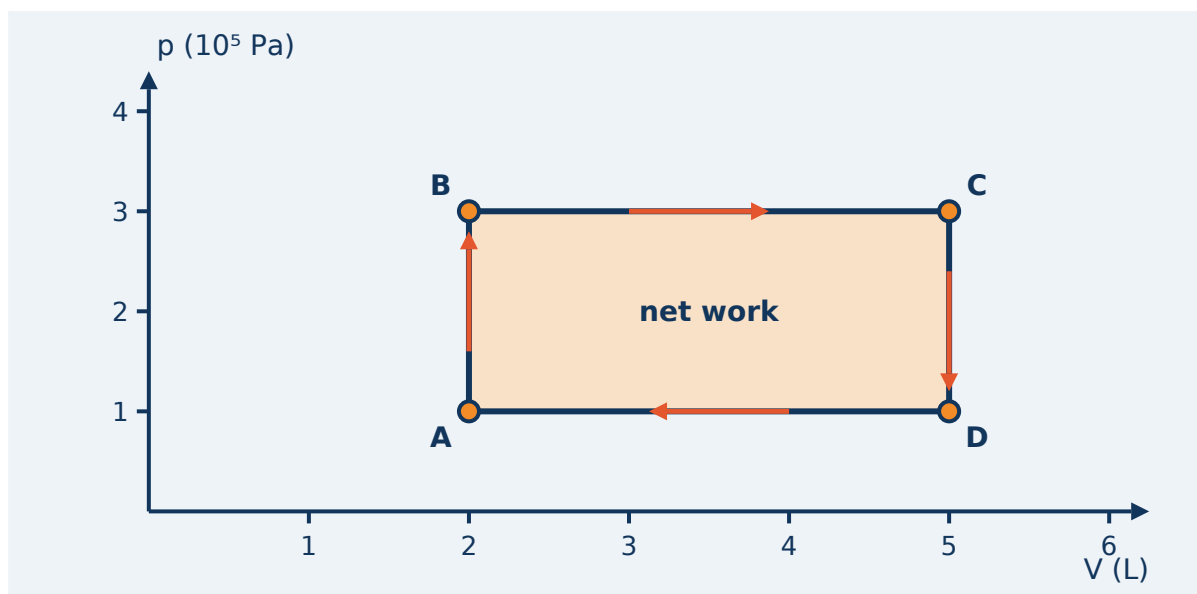
$$e = W_{\text{net}}/Q_H = 1 - Q_C/Q_H \quad e_{\text{max}} = 1 - T_C/T_H$$

$e$  is the efficiency (no unit),  $W_{\text{net}}$  is the work per cycle (J),  $Q_H$  and  $Q_C$  are the heats absorbed and rejected (J), and  $T_H$ ,  $T_C$  are the reservoir temperatures in kelvin.

### Worked example

An engine absorbs 800 J per cycle and rejects 560 J. Find  $W_{\text{net}}$  and  $e$ . Then find the maximum efficiency between 500 K and 300 K.

$W_{\text{net}} = 800 - 560 = \mathbf{240 \text{ J}}$ ;  $e = 240/800 = \mathbf{0.30}$ . Maximum:  $1 - 300/500 = \mathbf{0.40}$ , so the engine is allowed.



A clockwise cycle: the enclosed area is the net work done by the gas per cycle.

### Worked example

The cycle in the figure runs between  $1.0 \times 10^5 \text{ Pa}$  and  $3.0 \times 10^5 \text{ Pa}$  and between 2.0 L and

5.0 L. Find the net work per cycle.

Area =  $\Delta p \times \Delta V = 2.0 \times 10^5 \times 3.0 \times 10^{-3} = \mathbf{600 \text{ J}}$ , done by the gas because the loop is clockwise.

## What are entropy and the second law?

Entropy measures how many ways energy can be spread among the particles. When heat  $Q$  flows reversibly at temperature  $T$ , the entropy change is  $\Delta S = Q/T$ . The second law says the total entropy of an isolated system never decreases. That is why heat flows spontaneously from hot to cold, and why no engine can turn all the heat it takes in into work.

### Lina's tip

On any PV diagram, label each corner with its  $p$  and  $V$ , then find the temperature trend from  $pV$ . If  $pV$  rises,  $T$  rises and  $\Delta U$  is positive. This check settles most sign questions.

## How do AP questions test these ideas?

Expect to read a PV diagram, apply  $\Delta U = Q + W$  to each step, and justify a claim in a paragraph, such as why an adiabatic compression raises the temperature: work is done on the gas,  $Q = 0$ , so  $\Delta U$  is positive. You may also be asked to evaluate an engine claim by comparing its efficiency with the Carnot limit and by checking the total entropy change.

### Key points to remember

- $\Delta U = Q + W$ , with  $W$  the work done on the gas
- Constant pressure:  $W = -p\Delta V$ ; area under the PV curve gives the work
- Monatomic ideal gas:  $\Delta U = (3/2)nR\Delta T$ ; a full cycle has  $\Delta U = 0$
- Engine efficiency  $e = W_{\text{net}}/Q_{\text{H}} = 1 - Q_{\text{C}}/Q_{\text{H}}$ , and  $e_{\text{max}} = 1 - T_{\text{C}}/T_{\text{H}}$
- Entropy change  $\Delta S = Q/T$ ; total entropy never decreases

## Heat, work and engines on PV diagrams: frequently asked questions

### Why is work negative when a gas expands?

In the AP convention  $W$  is the work done on the gas. An expanding gas pushes on its surroundings, so energy leaves the gas and  $W$  is negative. A compression gives positive  $W$ .

### What is zero in an isothermal process?

The change in internal energy of an ideal gas, because  $U$  depends only on temperature. The first law then gives  $Q = -W$ : all heat entering becomes work done by the gas.

### What does an adiabatic process mean?

No heat flows into or out of the gas, so  $Q = 0$  and  $\Delta U = W$ . A quick compression raises the

temperature, and a quick expansion lowers it.

**Why can no engine be 100 % efficient?**

The second law requires some heat  $Q_C$  to be rejected to a colder reservoir. Turning all the heat into work would reduce the total entropy, which never happens in nature.

**How do you find the net work of a cycle on a PV diagram?**

Find the area enclosed by the loop. A clockwise loop means positive net work done by the gas, and an anticlockwise loop means net work done on the gas, as in a refrigerator.