

# Forces, friction and motion on slopes

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

**Quick answer:** Newton's second law states that the resultant force equals mass times acceleration,  $\Sigma F = ma$ . Friction opposes sliding with  $f = \mu N$ , and on a slope of angle  $\theta$  the normal reaction is  $mg \cos \theta$  while  $mg \sin \theta$  pulls the object down the slope. For connected bodies the acceleration is shared and the tension links the equations.

**Level:** Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB A.2

## What does Newton's second law say?

The **resultant force** on an object equals its mass multiplied by its acceleration. In symbols,  $\Sigma F = ma$ . The resultant is found by adding all the forces as vectors, so friction, tension, weight and the normal reaction must all be included with their directions.

### Formula

$\Sigma F = ma$ .  $\Sigma F$  is the resultant force in newtons (N),  $m$  is mass in kilograms (kg) and  $a$  is acceleration in  $m/s^2$ . Weight  $W = mg$  with  $g = 9.81 \text{ N/kg}$ .

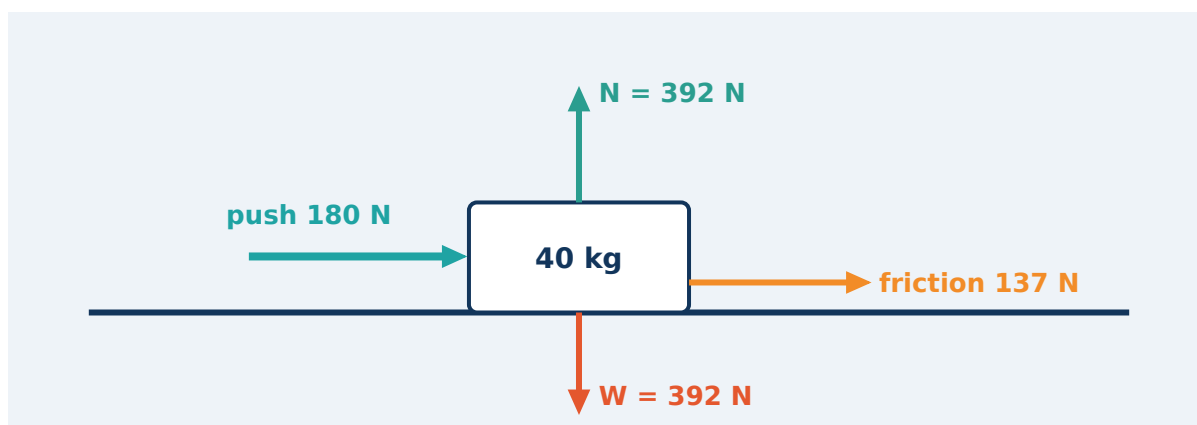
## How does friction work?

Friction is a contact force that opposes the relative motion, or the tendency to move, between two surfaces. Static friction can adjust itself up to a limit. Once the object slides, the friction has a nearly constant value.

### Definition

The coefficient of friction  $\mu$  is the ratio of the friction force to the normal reaction force  $N$ . For sliding,  $f = \mu_k N$ . For the largest value before sliding starts,  $f_{\max} = \mu_s N$ . Static friction is at most  $\mu_s N$ .

| Pair of surfaces        | Typical $\mu$ | Comment              |
|-------------------------|---------------|----------------------|
| Rubber tyre on dry road | 0.7 to 0.9    | good grip            |
| Wood on wood            | 0.3 to 0.5    | depends on roughness |
| Steel on steel, dry     | about 0.5     | varies with polish   |
| Ice on ice              | about 0.03    | very slippery        |



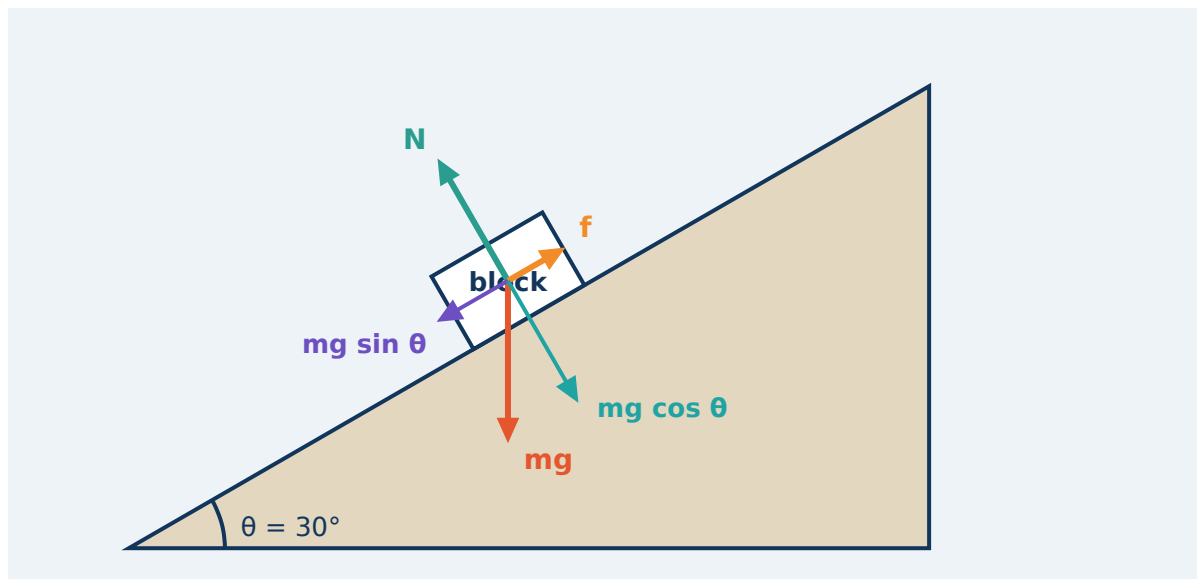
A crate pushed along a rough floor.  $N$  equals  $mg$  on a level surface.

### Worked example 1

A 40 kg crate is pushed with 180 N along a floor with  $\mu_k = 0.35$ .  $N = mg = 40 \times 9.81 = 392$  N. Friction  $f = 0.35 \times 392 = 137$  N. Resultant  $= 180 - 137 = 43$  N.  $a = 43 \div 40 = \mathbf{1.07 \text{ m/s}^2}$  in the direction of the push.

## How do you analyse motion on a slope?

On a slope of angle  $\theta$  the weight  $mg$  is split into a component  $mg \sin \theta$  pulling the object down the slope and a component  $mg \cos \theta$  pressing it into the surface. The normal reaction balances the second component, so  $N = mg \cos \theta$ , which is smaller than  $mg$ .



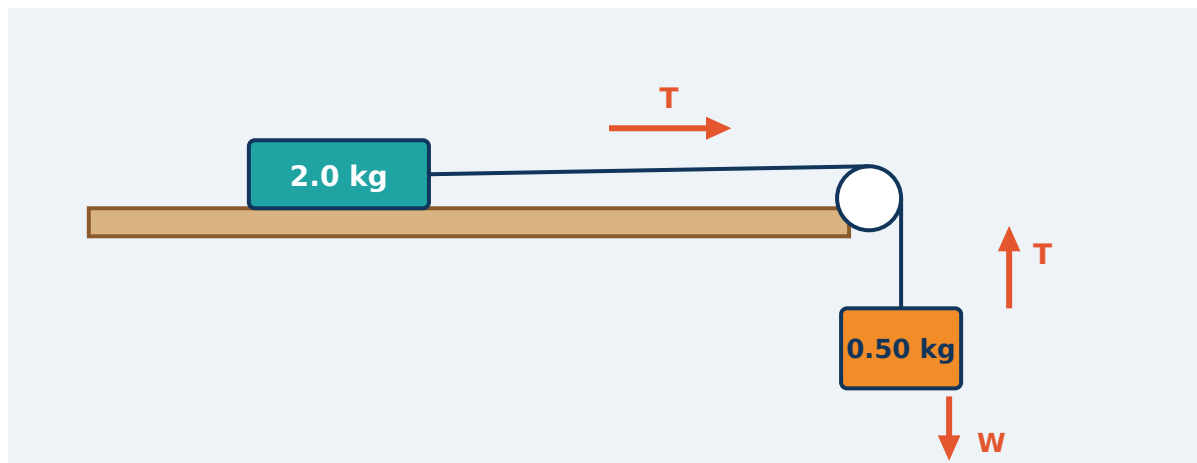
Forces on a block sliding down a slope.

### Worked example 2

A 5.0 kg box slides down a  $30^\circ$  slope with  $\mu_k = 0.20$ . Down the slope:  $mg \sin \theta = 5.0 \times 9.81 \times 0.500 = 24.5$  N.  $N = 5.0 \times 9.81 \times 0.866 = 42.5$  N and  $f = 0.20 \times 42.5 = 8.5$  N. Resultant  $= 24.5 - 8.5 = 16.0$  N, so  $a = 16.0 \div 5.0 = \mathbf{3.2 \text{ m/s}^2}$ . Mass cancels:  $a = g(\sin \theta - \mu \cos \theta)$ .

## How do you treat connected bodies?

If two bodies are joined by a taut string, they have the same acceleration. Write  $\Sigma F = ma$  for each body separately, then solve the two equations together. The tension in the string is the same throughout a light string over a smooth pulley.



A trolley pulled by a hanging mass over a pulley.

### Worked example 3

The 2.0 kg trolley and the 0.50 kg hanging mass are on a smooth table. For the whole system:  $0.50 \times 9.81 = (2.0 + 0.50) \times a$ , so  $a = 4.905 \div 2.5 = \mathbf{1.96 \text{ m/s}^2}$ . For the trolley:  $T = 2.0 \times 1.962 = \mathbf{3.92 \text{ N}}$ , which is less than the weight of the hanging mass.

### Common mistake

Students write  $N = mg$  on a slope and put friction as  $\mu mg$ . The normal reaction is  $mg \cos \theta$ , so friction is  $\mu mg \cos \theta$ . Also, friction opposes the motion; it does not always act backwards, it acts against the sliding direction.

### Lina's tip

Draw the free-body diagram first and choose axes along and perpendicular to the slope. Write one equation for each axis, and the algebra almost solves itself.

### Key points to remember

- $\Sigma F = ma$ ; weight  $W = mg$ ,  $g = 9.81 \text{ N/kg}$
- Friction  $f \leq \mu N$ , sliding  $f = \mu k N$
- On a slope:  $N = mg \cos \theta$ , driving force  $mg \sin \theta$
- Slope acceleration:  $a = g(\sin \theta - \mu k \cos \theta)$
- Connected bodies share the acceleration; tension links the equations

## Forces, friction and motion on slopes: frequently asked

## questions

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### What is the difference between static and kinetic friction?

Static friction acts when there is no sliding and can take any value up to a maximum of  $\mu_s N$ . Kinetic friction acts when the surfaces slide and is roughly constant at  $\mu_k N$ . Normally  $\mu_s$  is slightly larger than  $\mu_k$ .

### Why is the normal reaction smaller on a slope?

Only the component of the weight perpendicular to the surface,  $mg \cos \theta$ , presses the object into the slope. The surface balances this component. The component along the slope,  $mg \sin \theta$ , does not press on the surface.

### Does mass affect how fast an object slides down a slope?

For a given slope and coefficient of friction, no. Both the driving force and the friction are proportional to mass, so mass cancels:  $a = g(\sin \theta - \mu \cos \theta)$ . Heavy and light objects accelerate equally.

### How do you find the tension in a string joining two bodies?

First find the acceleration of the whole system. Then apply  $\Sigma F = ma$  to one body only, including the tension as the unknown force. The tension is the same at both ends of a light string over a smooth pulley.

### How can you measure the coefficient of static friction with a slope?

Raise one end of a board until the block just starts to slide. At that angle  $\mu_s = \tan \theta$ , because the friction limit equals the component of weight along the slope. The mass of the block is not needed.