

Friction, slopes, pulleys and springs

AP Physics 1 (algebra-based) · Lesson

Quick answer: Friction is the contact force that resists sliding. Static friction rises to a maximum of $\mu_s N$, and kinetic friction equals $\mu_k N$. On an incline the normal force is $mg \cos \theta$, and connected objects share one acceleration, so a single net-force equation per object solves the tension.

Level: AP Physics 1 (algebra-based) · **Curriculum:** AP P1 Unit 2

What is friction and how is it calculated?

Friction is the contact force parallel to a surface that resists sliding between two surfaces. It acts opposite to the way the surfaces would slide or are sliding, and it depends on how hard the surfaces are pressed together, not on how large the contact area is.

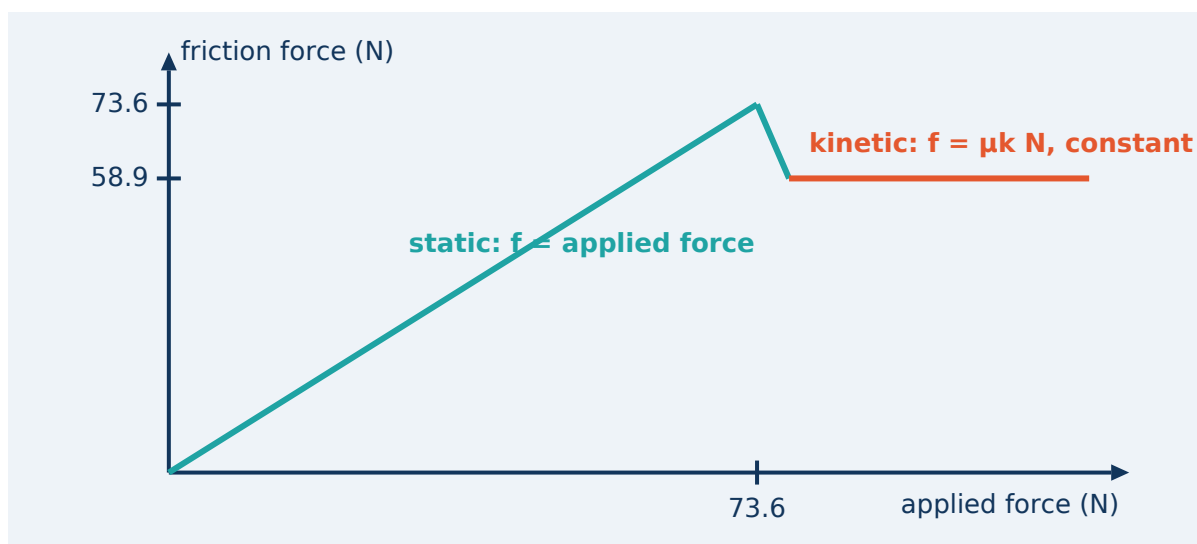
Definition

Static friction acts between surfaces that are not sliding; its size adjusts up to a maximum $f_{s,\max} = \mu_s N$. **Kinetic friction** acts between sliding surfaces and has the fixed size $f_k = \mu_k N$. The coefficient μ has no unit, and N is the normal force.

Situation	Friction force	Typical μ
rubber on dry concrete	$\mu_s \approx 0.8$	0.6 to 1.0
wood on wood	$\mu_s \approx 0.4$	0.25 to 0.5
steel on ice	$\mu_k \approx 0.03$	0.02 to 0.05

Worked example

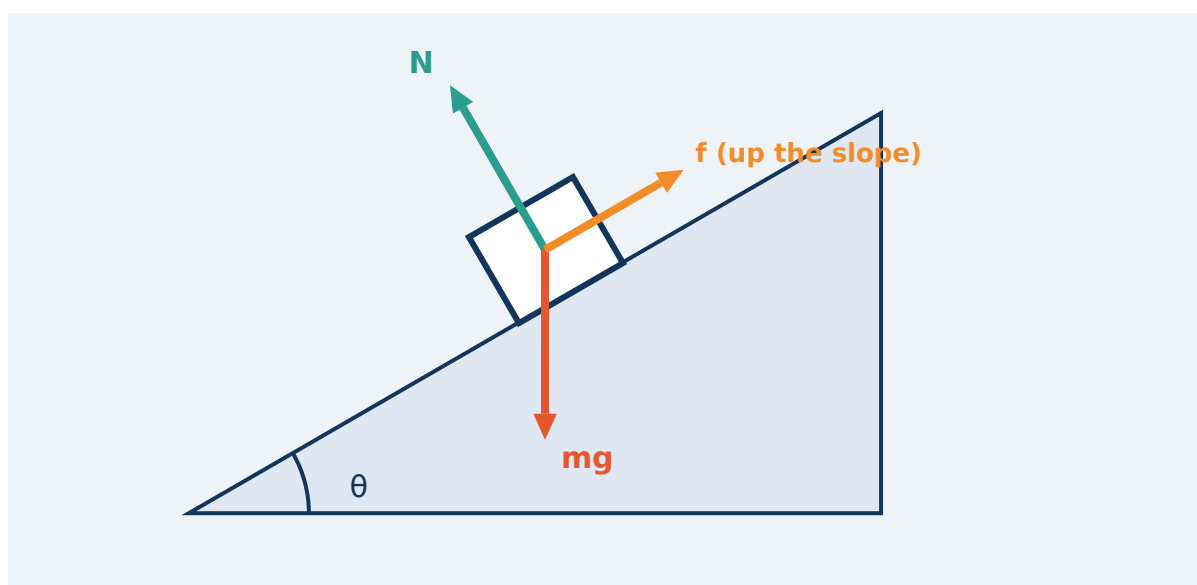
Wanjiru pushes a 15 kg crate across a floor with $\mu_s = 0.50$ and $\mu_k = 0.40$. $N = mg = 147 \text{ N}$. Maximum static friction = $0.50 \times 147 = \mathbf{73.6 \text{ N}}$. At a push of 50 N the crate does not move and the friction is 50 N.
At 80 N it slides: $f_k = 0.40 \times 147 = \mathbf{58.9 \text{ N}}$, so $a = (80 - 58.9) \div 15 = \mathbf{1.4 \text{ m/s}^2}$.



A 15 kg crate with $\mu_s = 0.50$ and $\mu_k = 0.40$ on a level floor.

How do you solve an incline problem?

Tilt the axes: x along the slope, y perpendicular to it. Only the weight needs resolving: its component along the slope is $mg \sin \theta$ and into the slope is $mg \cos \theta$. There is no motion perpendicular to the surface, so $N = mg \cos \theta$.



Weight is the only force not along or perpendicular to the slope; resolve it into $mg \sin \theta$ along the slope and $mg \cos \theta$ into the slope.

Formula

$a = g (\sin \theta - \mu_k \cos \theta)$ for sliding down

a = acceleration down the slope (m/s^2); $g = 9.81 \text{ m/s}^2$; θ = slope angle; μ_k = kinetic coefficient. A block just starts to slide when $\tan \theta = \mu_s$.

Worked example

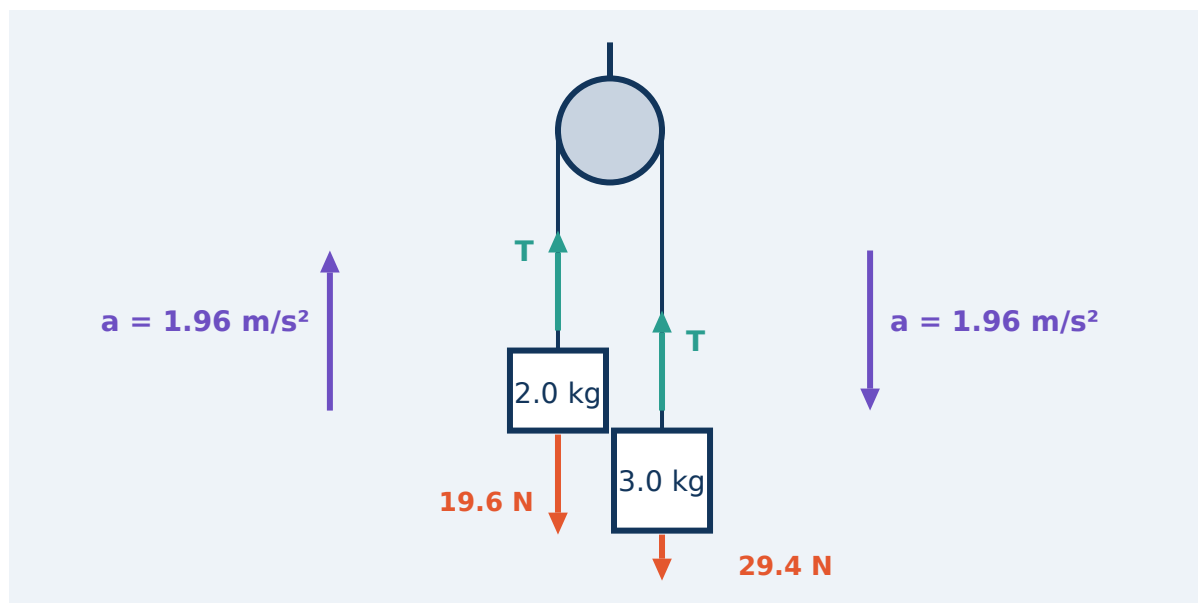
A 10 kg box slides down a 30° ramp with $\mu_k = 0.20$.

$$a = 9.81 \times (\sin 30^\circ - 0.20 \times \cos 30^\circ) = 9.81 \times (0.500 - 0.173) = \mathbf{3.2 \text{ m/s}^2}.$$

$$\text{Friction} = 0.20 \times (10 \times 9.81 \times \cos 30^\circ) = \mathbf{17 \text{ N}}$$
 up the slope.

How do you analyse connected objects and springs?

Objects joined by a light string share the same acceleration size and, over a frictionless pulley, the same tension. Treat the whole system with one equation for the total mass, then isolate one object to find the tension. For a spring the force is $F = kx$, where k is the spring constant (N/m) and x the stretch or compression from the natural length (m).



Same string tension on both masses (light string, frictionless pulley); the accelerations have equal size.

Worked example

In an Atwood machine $m_1 = 2.0 \text{ kg}$ and $m_2 = 3.0 \text{ kg}$.

System: $(m_2 - m_1)g = (m_1 + m_2)a$, so $a = 1.0 \times 9.81 \div 5.0 = \mathbf{1.96 \text{ m/s}^2}$.

Light mass: $T - m_1g = m_1a$, so $T = 2.0 \times (9.81 + 1.96) = \mathbf{23.5 \text{ N}}$.

Common mistake

Students often use the weight mg instead of $mg \cos \theta$ in $f = \mu N$ on a slope, or assume the tension equals the weight of a hanging mass. The tension equals the weight only when the mass is not accelerating.

Lina's tip

For connected objects, write one sentence per object: "net force on this object = its mass $\times$ a". Add the equations so the tension disappears, then put the answer back to find T .

Key points to remember

- Static friction: $f \leq \mu_s N$; kinetic friction: $f = \mu_k N$.
- On a slope: $N = mg \cos \theta$ and the weight component along it is $mg \sin \theta$.
- Sliding down: $a = g (\sin \theta - \mu_k \cos \theta)$.

- Atwood machine: $a = (m_2 - m_1)g \div (m_1 + m_2)$.
- Spring force: $F = kx$, with k in N/m.

Friction, slopes, pulleys and springs: frequently asked questions

Does friction depend on the contact area?

In the simple model used in AP Physics 1, no. Friction depends on the normal force and on the surfaces through the coefficient μ , not on how large the area of contact is.

Why is the normal force less than mg on a slope?

The surface only has to balance the part of the weight pressing into it. That is the component $mg \cos \theta$, which gets smaller as the slope gets steeper.

How do you find the tension in a pulley problem?

Apply $\Sigma F = ma$ to the whole system to find the acceleration, then apply it to one mass. For a hanging mass the tension is $T = m(g - a)$ when it accelerates downward.

What does the spring constant tell you?

The spring constant k in N/m is the force needed per metre of stretch. A stiffer spring has a larger k , and $F = kx$ holds while the spring stays within its elastic limit.

How can you measure μ_s with a tilting board?

Raise the board slowly until the block just slides and measure the angle θ . At that point $\mu_s = \tan \theta$, because the weight component along the slope equals the maximum static friction.