

Flux, Gauss's law and continuous charge

AP Physics C (Mechanics and E&M, calculus-based) · Lesson

Quick answer: Electric flux is $\Phi = \oint \mathbf{E} \cdot d\mathbf{A}$, the field passing through a surface. Gauss's law states that the net flux through a closed surface equals the enclosed charge over ϵ_0 . With enough symmetry, it gives the field of spheres, lines and sheets quickly.

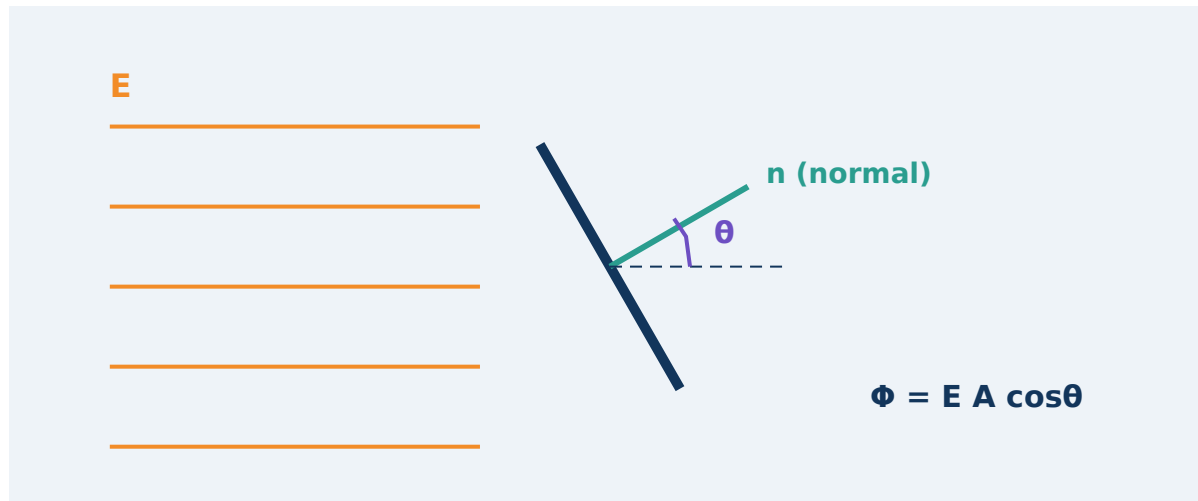
Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 8

What is electric flux?

Electric flux measures how much electric field passes through a surface. Picture field lines as streams of water and a surface as a net held in the flow. The more lines that cross the net, the larger the flux, and a net held edge-on catches none.

Definition

The electric flux through a surface is $\Phi = \int \mathbf{E} \cdot d\mathbf{A}$, in $\text{N m}^2/\text{C}$. For a uniform field through a flat area A , $\Phi = E A \cos\theta$, where θ is the angle between $\mathbf{E}$ and the normal to the surface. For a closed surface the normal points outward.



Flux through a flat surface: $\Phi = E A \cos\theta$, where θ is the angle between $\mathbf{E}$ and the normal $\mathbf{n}$.

What does Gauss's law say?

Key law

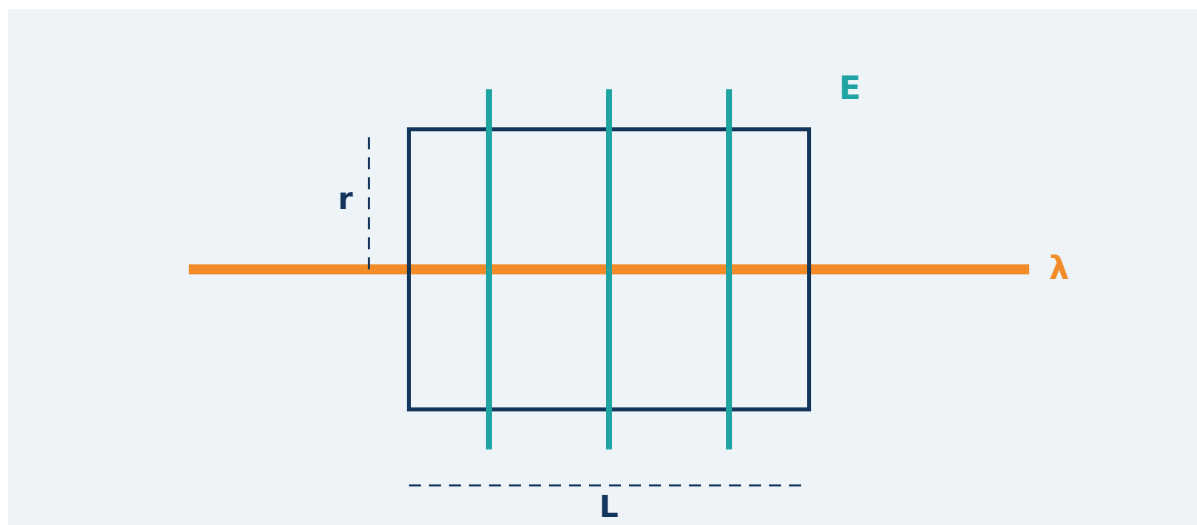
Gauss's law: the net flux through any closed surface equals the enclosed charge divided by ϵ_0 . $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}} / \epsilon_0$, where $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$ and $k = 1/(4\pi\epsilon_0) = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$.

The law holds for every closed surface and every charge arrangement. Charges outside the

surface contribute to E on the surface, but their field lines enter and leave, so they add nothing to the net flux. Gauss's law is most useful when symmetry lets E be constant over the surface and perpendicular to it, so that the integral becomes E times an area.

How do you find the field of a continuous charge distribution?

Two methods exist. The direct method divides the charge into pieces dq , each giving $dE = k dq / r^2$, and integrates the components. For a ring of charge Q and radius R , the perpendicular parts cancel on the axis, leaving $E = kQx / (x^2 + R^2)^{3/2}$. The second method uses Gauss's law and symmetry.



Gaussian cylinder around a line charge: flux passes only through the curved side, area $2\pi rL$.

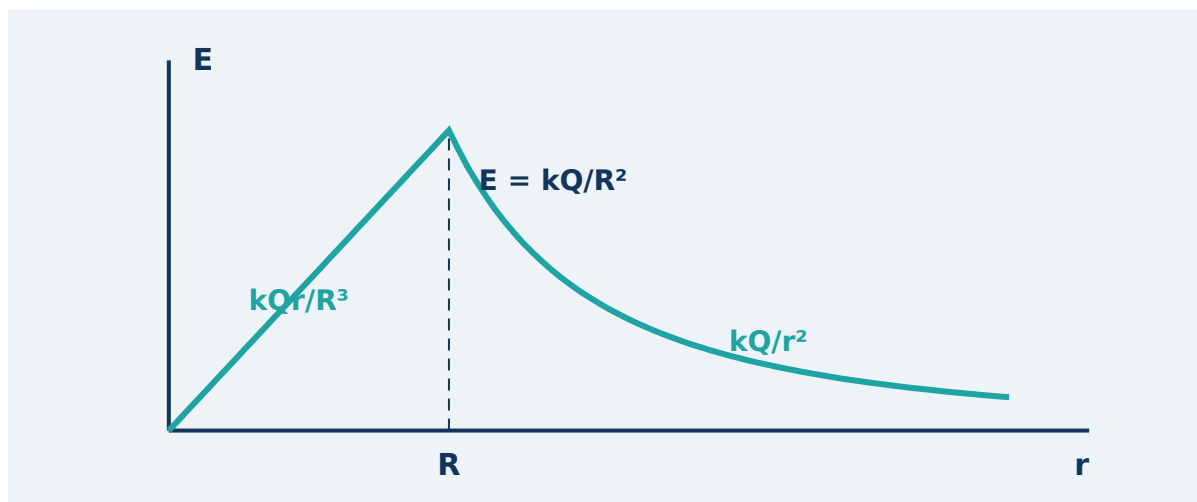
For a long line of charge, choose a coaxial cylinder of radius r and length L . Flux leaves only through the curved side: $E(2\pi rL) = \lambda L / \epsilon_0$, which gives $E = \lambda / (2\pi\epsilon_0 r)$.

Charge distribution	Gaussian surface	Result for E
Point charge or sphere (shell or solid)	Concentric sphere, area $4\pi r^2$	$E = kQ_{\text{enc}}/r^2$
Long line of charge λ	Coaxial cylinder, side area $2\pi rL$	$E = \lambda/(2\pi\epsilon_0 r)$
Large flat sheet σ	Pillbox through the sheet, 2 end faces	$E = \sigma/(2\epsilon_0)$
Surface of a conductor	Small pillbox, one face outside	$E = \sigma/\epsilon_0$

Worked example 1

A long wire carries a uniform charge of $\lambda = 2.0 \mu\text{C/m}$. At $r = 0.50 \text{ m}$ the field is $E = \lambda / (2\pi\epsilon_0 r) = 2.0 \times 10^{-6} / (2\pi \times 8.85 \times 10^{-12} \times 0.50) = \mathbf{7.2 \times 10^4 \text{ N/C}}$, directed radially away from the wire. Check: $2k\lambda/r = 2 \times 8.99 \times 10^9 \times 2.0 \times 10^{-6} / 0.50 = 7.2 \times 10^4 \text{ N/C}$.

What is the field of a charged sphere?



Uniformly charged insulating sphere: $E \propto r$ inside and $E \propto 1/r^2$ outside.

For a uniformly charged solid insulating sphere of radius R , use a concentric spherical surface of radius r . Outside ($r \geq R$) all of Q is enclosed, so $E = kQ/r^2$. Inside the sphere (r less than R) the enclosed charge is $Q (r/R)^3$, so $E (4\pi r^2) = Q r^3 / (R^3 \epsilon_0)$, giving $E = kQ r / R^3$. The field peaks at the surface.

Worked example 2

A solid insulating sphere has $Q = +6.0 \text{ nC}$ and $R = 0.10 \text{ m}$. At $r = 0.05 \text{ m}$, $E = kQr/R^3 = 8.99 \times 10^9 \times 6.0 \times 10^{-9} \times 0.05 / (0.10)^3 = \mathbf{2.7 \times 10^3 \text{ N/C}}$. At $r = 0.20 \text{ m}$, $E = kQ/r^2 = 8.99 \times 10^9 \times 6.0 \times 10^{-9} / 0.040 = \mathbf{1.3 \times 10^3 \text{ N/C}}$. At the surface the field is $5.4 \times 10^3 \text{ N/C}$.

Conductors behave differently. In electrostatic equilibrium the field inside a conductor is zero, all net charge sits on the outer surface, and the field just outside is σ/ϵ_0 perpendicular to the surface.

Common mistake

Do not say that the field at a point on a Gaussian surface comes only from the enclosed charge. The flux depends only on the enclosed charge, but the field E at each point is produced by all charges, inside and outside.

Lina's tip

Before using Gauss's law, ask: can I draw a surface where E has the same size everywhere on it and is perpendicular or parallel to it? If not, the law is true but will not help, and you should integrate dE directly.

Key points to remember

- Flux: $\Phi = \oint \mathbf{E} \cdot d\mathbf{A} = E A \cos\theta$ for a uniform field ($\text{N m}^2/\text{C}$).
- Gauss's law: $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}} / \epsilon_0$, with $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$.
- Line of charge: $E = \lambda/(2\pi\epsilon_0 r)$; infinite sheet: $E = \sigma/(2\epsilon_0)$; conductor surface: $E = \sigma/\epsilon_0$.
- Solid insulating sphere: $E = kQr/R^3$ inside, $E = kQ/r^2$ outside.
- Inside a conductor in equilibrium $E = 0$ and the net charge is on its surface.

Flux, Gauss's law and continuous charge: frequently asked questions

What are the units of electric flux?

Electric flux is measured in $\text{N m}^2/\text{C}$, which is the same as V m . The unit comes from multiplying a field in N/C by an area in m^2 . Flux through a closed surface equals Q/ϵ_0 and so tells you the enclosed charge.

Why is the flux zero for a charge outside the surface?

Every field line from an outside charge that enters the closed surface must leave it again. The inward flux, which is negative, equals the outward flux, so the net flux is zero. The charge still affects E at points on the surface.

When can Gauss's law be used to find E ?

When symmetry makes E constant in size and either perpendicular or parallel to each part of a chosen surface. Spherical, cylindrical and planar symmetry satisfy this. Without symmetry, integrate $dE = k dq/r^2$ directly.

Why is the field inside a conductor zero?

Free charges move until the net force on each is zero. If E were non-zero inside, charges would keep moving. In equilibrium the field vanishes in the bulk, so any extra charge sits on the surface.

How does the field of a sheet differ from that of a point charge?

A large uniform sheet gives $E = \sigma/(2\epsilon_0)$, which does not depend on distance. A point charge's field falls as $1/r^2$. The sheet field is constant because the area of field lines is constant, unlike the spreading area of a sphere.