

Flux, Gauss's law and continuous charge

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

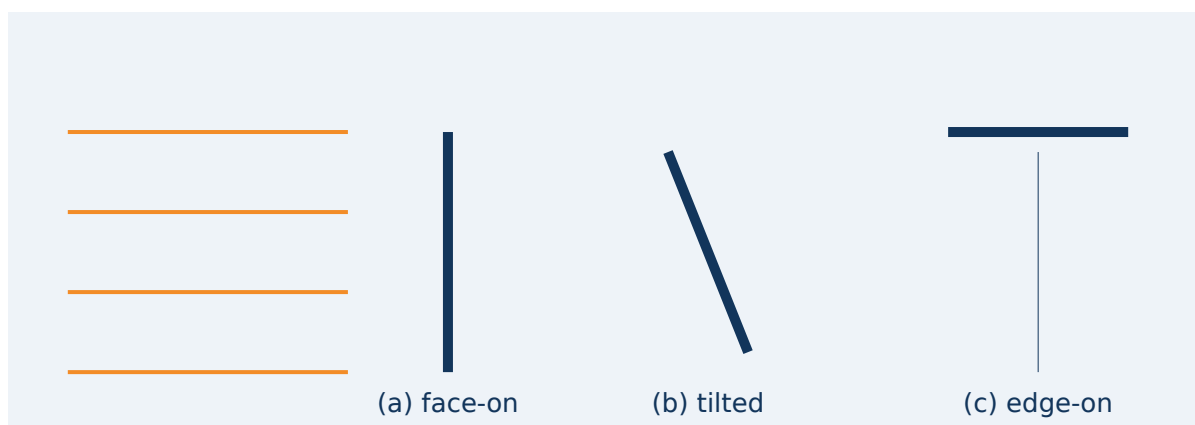
Name: _____ Date: _____ Class: _____

These problems train the calculus-based treatment of electric fields: building E from continuous charge by integration, computing flux, and applying Gauss's law with spherical, cylindrical and planar symmetry. Contexts include wires, plates, shells and cubes.

Level 1 problems use a single relation, level 2 problems combine superposition with symmetry, and level 3 problems need integration, non-uniform charge or conductors, as in AP Physics C: Electricity and Magnetism unit 8.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 8

Problem 1 Flux of a uniform field (easy)



Problem 1: the same surface in three orientations.

A uniform field $E = 500 \text{ N/C}$ passes through a square surface of area 0.20 m^2 (figure). Calculate the flux when

1. the surface is face-on to the field (normal parallel to E)
2. the normal makes 60° with E
3. the surface is edge-on.

Problem 2 Charge from flux (easy)

The net outward flux through a closed surface is $3.0 \times 10^3 \text{ N m}^2/\text{C}$. Calculate the charge enclosed.

Problem 3 Flux from a point charge (easy)

A point charge of $+4.0 \text{ nC}$ sits at the centre of a sphere of radius 0.10 m . Calculate the flux through the sphere and say what happens if the sphere radius is doubled.

Problem 4 Field of a point charge (easy)

Calculate the electric field 0.30 m from a point charge of +5.0 nC and state its direction.

Problem 5 Net enclosed charge (easy)

A closed surface encloses charges of +3.0 nC and -1.0 nC. A further +8.0 nC charge lies outside it.

1. Calculate the net flux through the surface.
2. State the effect of the outside charge.

Problem 6 Charge densities (easy)

1. A rod 0.60 m long carries 3.0 nC uniformly. Calculate λ .
2. A plate of area 0.30 m² carries 12 nC uniformly on one face. Calculate σ .

Problem 7 Charged ring on its axis (medium)

A thin ring of radius 0.12 m carries a uniform charge of +5.0 nC. Calculate the field on its axis at 0.16 m from the centre, and state the field at the centre.

Problem 8 Long wire (medium)

A long straight wire carries a uniform charge of 3.0 nC/m. Calculate the field 0.40 m from it.

Problem 9 Large sheet (medium)

A large flat insulating sheet carries a uniform surface charge of +4.0 nC/m².

1. Calculate the field 0.10 m from it.
2. Calculate the field 1.0 m from it.

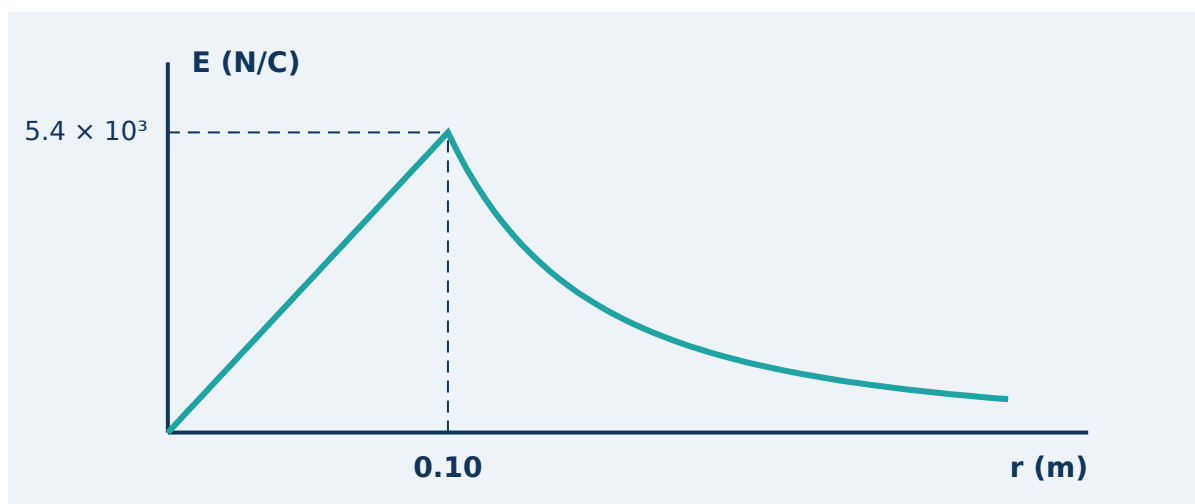
Problem 10 Thin spherical shell (medium)

A thin spherical shell of radius 0.20 m carries a charge of -12 nC. Calculate the field at (a) $r = 0.10$ m and (b) $r = 0.50$ m.

Problem 11 Solid insulating sphere (medium)

A solid insulating sphere of radius 0.15 m carries a uniform +10 nC. Calculate the field at $r = 0.10$ m and at $r = 0.30$ m.

Problem 12 Reading an E-r graph (medium)

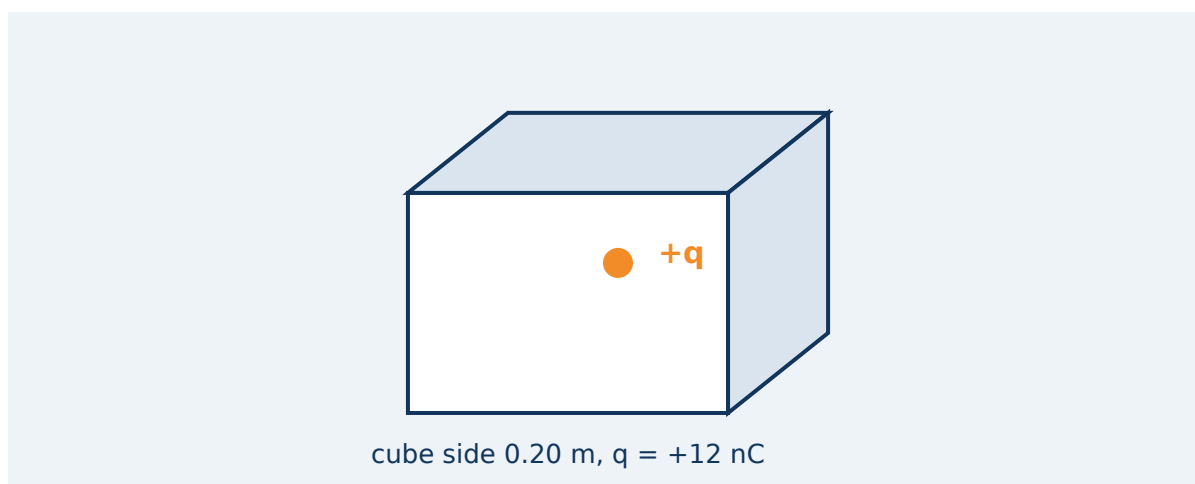


Problem 12: E against r for a uniformly charged sphere.

The graph shows the field of a uniformly charged solid insulating sphere.

1. State the radius of the sphere.
2. Calculate the charge on it.
3. Calculate the field at $r = 0.20$ m.

Problem 13 Charge at the centre of a cube (medium)



Problem 13: a point charge at the centre of a cube.

A charge of $+12$ nC sits at the centre of a cube (figure). Calculate the total flux and the flux through one face.

Problem 14 Spot the error (medium)

A student writes: 'In Gauss's law the field E on the surface is the field of the enclosed charge only, so a charge outside the surface has no effect.'

1. Identify the error.
2. State the correct statement.

Problem 15 Field between charged plates (medium)

Two large parallel plates carry equal and opposite surface charge densities of $\pm 2.0 \times 10^{-7} \text{ C/m}^2$.

1. Calculate the field between them.
2. Calculate the acceleration of an electron placed there.

Problem 16 Maximum field on a ring's axis (hard)

A ring of radius $R = 0.12 \text{ m}$ carries $+5.0 \text{ nC}$, and $E(x) = kQx / (x^2 + R^2)^{(3/2)}$.

1. Show that the field is greatest at $x = R/\sqrt{2}$.
2. Calculate the maximum field.

Problem 17 Field of a finite rod (hard)

A rod of length 0.40 m carries $+8.0 \text{ nC}$ uniformly. A point P lies on the line of the rod, 0.10 m beyond one end.

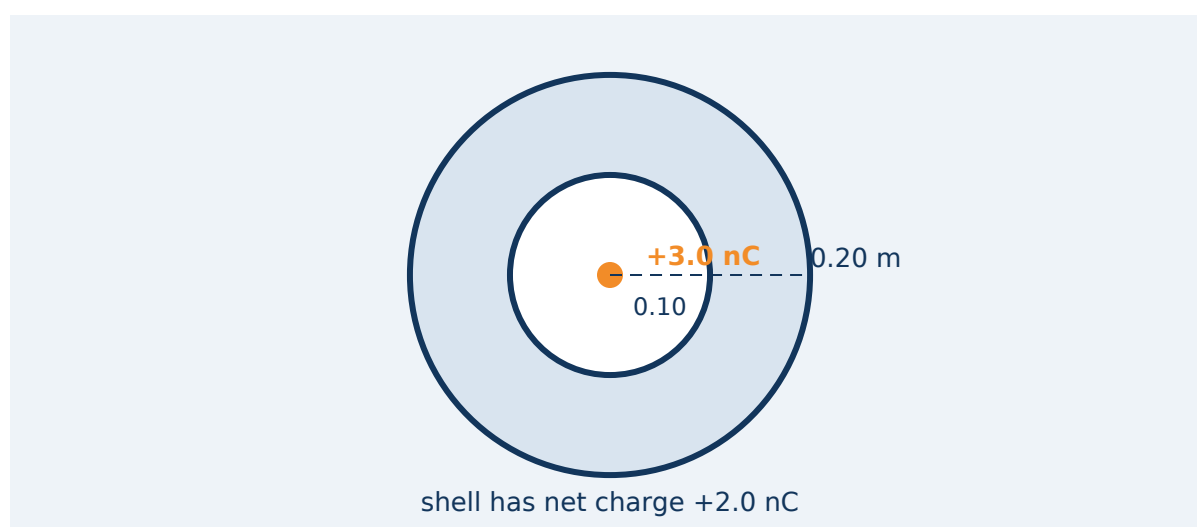
1. Set up and evaluate the integral for E at P.
2. Calculate E.

Problem 18 Non-uniform sphere (hard)

A sphere of radius $R = 0.20 \text{ m}$ has volume charge density $\rho = A r$ with $A = 3.0 \times 10^{-6} \text{ C/m}^4$.

1. Calculate the total charge.
2. Use Gauss's law to find E at $r = 0.10 \text{ m}$.
3. Calculate E at $r = 0.50 \text{ m}$.

Problem 19 Charge inside a conducting shell (hard)



Problem 19: a point charge inside a conducting shell.

A $+3.0 \text{ nC}$ charge sits at the centre of a thick conducting shell (inner radius 0.10 m , outer radius 0.20 m) whose net charge is $+2.0 \text{ nC}$.

1. State the charge on the inner and outer surfaces.

2. Calculate E at $r = 0.05$ m and at $r = 0.30$ m.
3. State E inside the metal.

Problem 20 Long charged cylinder (hard)

A very long solid insulating cylinder of radius 0.050 m has uniform volume charge density $\rho = +2.0 \mu\text{C}/\text{m}^3$.

1. Use Gauss's law to find E inside at $r = 0.030$ m.
2. Calculate E outside at $r = 0.10$ m.