

Flux, Gauss's law and continuous charge

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet answers

Each answer shows the Gaussian surface or the integral first, then the substitution and the result with its unit in bold. Use $k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$. Check signs and directions, not only magnitudes.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:E&M Unit 8

Problem 1 Flux of a uniform field

$$\Phi = E A \cos\theta.$$

- $\Phi = 500 \times 0.20 \times 1 = \mathbf{1.0 \times 10^2 \text{ N m}^2/\text{C}}.$
- $\Phi = 500 \times 0.20 \times \cos 60^\circ = \mathbf{50 \text{ N m}^2/\text{C}}.$
- The normal is at 90° , so $\Phi = \mathbf{0}.$

Problem 2 Charge from flux

$Q = \epsilon_0 \Phi = 8.85 \times 10^{-12} \times 3.0 \times 10^3 = \mathbf{2.7 \times 10^{-8} \text{ C}}$ (27 nC), positive because the flux is outward.

Problem 3 Flux from a point charge

$\Phi = q/\epsilon_0 = 4.0 \times 10^{-9} / 8.85 \times 10^{-12} = \mathbf{4.5 \times 10^2 \text{ N m}^2/\text{C}}.$ Doubling the radius changes nothing, because only the enclosed charge matters.

Problem 4 Field of a point charge

$E = kq/r^2 = 8.99 \times 10^9 \times 5.0 \times 10^{-9} / 0.30^2 = \mathbf{5.0 \times 10^2 \text{ N/C}},$ directed radially away from the positive charge.

Problem 5 Net enclosed charge

- $Q_{\text{enc}} = +2.0 \text{ nC},$ so $\Phi = 2.0 \times 10^{-9} / 8.85 \times 10^{-12} = \mathbf{2.3 \times 10^2 \text{ N m}^2/\text{C}}.$
- The outside charge changes E on the surface but adds **no net flux**.

Problem 6 Charge densities

- $\lambda = Q/L = 3.0 \times 10^{-9} / 0.60 = \mathbf{5.0 \times 10^{-9} \text{ C/m}}.$
- $\sigma = Q/A = 12 \times 10^{-9} / 0.30 = \mathbf{4.0 \times 10^{-8} \text{ C/m}^2}.$

Problem 7 Charged ring on its axis

$E = kQx / (x^2 + R^2)^{3/2}$. $x^2 + R^2 = 0.0256 + 0.0144 = 0.0400$, and $0.0400^{3/2} = 0.00800$. $E = 8.99 \times 10^9 \times 5.0 \times 10^{-9} \times 0.16 / 0.00800 = \mathbf{9.0 \times 10^2 \text{ N/C}}$ along the axis, away from the ring. At the centre, $E = \mathbf{0}$ by symmetry.

Problem 8 Long wire

$E = \lambda / (2\pi\epsilon_0 r) = 2k\lambda / r = 2 \times 8.99 \times 10^9 \times 3.0 \times 10^{-9} / 0.40 = \mathbf{1.3 \times 10^2 \text{ N/C}}$, radially outward.

Problem 9 Large sheet

$E = \sigma / (2\epsilon_0) = 4.0 \times 10^{-9} / (2 \times 8.85 \times 10^{-12}) = \mathbf{2.3 \times 10^2 \text{ N/C}}$.

- At 0.10 m: $\mathbf{2.3 \times 10^2 \text{ N/C}}$, away from the sheet.
- At 1.0 m: **the same**, since the field of an infinite sheet does not depend on distance.

Problem 10 Thin spherical shell

- (a) Inside a uniform shell $Q_{\text{enc}} = 0$, so $E = \mathbf{0}$.
 (b) $E = kQ/r^2 = 8.99 \times 10^9 \times 12 \times 10^{-9} / 0.25 = \mathbf{4.3 \times 10^2 \text{ N/C}}$, directed inward.

Problem 11 Solid insulating sphere

At 0.10 m the enclosed charge is $10 \times (0.10/0.15)^3 = 2.96 \text{ nC}$, so $E = 8.99 \times 10^9 \times 2.96 \times 10^{-9} / 0.010 = \mathbf{2.7 \times 10^3 \text{ N/C}}$. At 0.30 m, $E = kQ/r^2 = 8.99 \times 10^9 \times 10 \times 10^{-9} / 0.090 = \mathbf{1.0 \times 10^3 \text{ N/C}}$.

Problem 12 Reading an E-r graph

- The peak is at the surface: $\mathbf{R = 0.10 \text{ m}}$.
- $E = kQ/R^2$, so $Q = E R^2 / k = 5.4 \times 10^3 \times 0.010 / 8.99 \times 10^9 = \mathbf{6.0 \times 10^{-9} \text{ C}}$.
- $E = kQ/r^2 = 8.99 \times 10^9 \times 6.0 \times 10^{-9} / 0.040 = \mathbf{1.3 \times 10^3 \text{ N/C}}$.

Problem 13 Charge at the centre of a cube

Total: $\Phi = q/\epsilon_0 = 12 \times 10^{-9} / 8.85 \times 10^{-12} = \mathbf{1.4 \times 10^3 \text{ N m}^2/\text{C}}$. By symmetry each of the six faces carries one sixth: $\mathbf{2.3 \times 10^2 \text{ N m}^2/\text{C}}$.

Problem 14 Spot the error

- E in $\oint \mathbf{E} \cdot d\mathbf{A}$ is the total field at each point, from all charges.
- Only the net flux depends on the enclosed charge alone**; an outside charge changes the local field but contributes zero net flux.

Problem 15 Field between charged plates

1. $E = \sigma/\epsilon_0 = 2.0 \times 10^{-7} / 8.85 \times 10^{-12} = \mathbf{2.3 \times 10^4 \text{ N/C}}$.
2. $a = eE/m = 1.6 \times 10^{-19} \times 2.26 \times 10^4 / 9.11 \times 10^{-31} = \mathbf{4.0 \times 10^{15} \text{ m/s}^2}$.

Problem 16 Maximum field on a ring's axis

1. $dE/dx = kQ[(x^2 + R^2) - 3x^2]/(x^2 + R^2)^{(5/2)} = 0$ gives $R^2 = 2x^2$, so $x = R/\sqrt{2} = \mathbf{0.085 \text{ m}}$.
2. $E_{\text{max}} = kQ(R/\sqrt{2})/(1.5R^2)^{(3/2)} = 0.385 kQ/R^2 = 0.385 \times 3.12 \times 10^3 = \mathbf{1.2 \times 10^3 \text{ N/C}}$.

Problem 17 Field of a finite rod

1. $\lambda = Q/L$. Element $dq = \lambda dx$ at distance x from P gives $dE = k\lambda dx/x^2$. $E = \int$ from a to $a+L$ of $k\lambda dx/x^2 = k\lambda(1/a - 1/(a+L)) = kQ/(a(a+L))$.
2. $E = 8.99 \times 10^9 \times 8.0 \times 10^{-9} / (0.10 \times 0.50) = \mathbf{1.4 \times 10^3 \text{ N/C}}$, directed away from the rod.

Problem 18 Non-uniform sphere

1. $Q = \int A r (4\pi r^2) dr$ from 0 to $R = \pi A R^4 = \pi \times 3.0 \times 10^{-6} \times 0.0016 = \mathbf{1.5 \times 10^{-8} \text{ C}}$.
2. $Q_{\text{enc}} = \pi A r^4$, so $E (4\pi r^2) = \pi A r^4/\epsilon_0$ and $E = A r^2/(4\epsilon_0) = 3.0 \times 10^{-6} \times 0.010 / (4 \times 8.85 \times 10^{-12}) = \mathbf{8.5 \times 10^2 \text{ N/C}}$.
3. $E = kQ/r^2 = 8.99 \times 10^9 \times 1.51 \times 10^{-8} / 0.25 = \mathbf{5.4 \times 10^2 \text{ N/C}}$.

Problem 19 Charge inside a conducting shell

1. Inside the metal $E = 0$, so the inner surface carries $\mathbf{-3.0 \text{ nC}}$ and the outer surface carries $\mathbf{+5.0 \text{ nC}}$ (net $+2.0 \text{ nC}$).
2. At 0.05 m , $E = kq/r^2 = 8.99 \times 10^9 \times 3.0 \times 10^{-9} / 0.0025 = \mathbf{1.1 \times 10^4 \text{ N/C}}$. At 0.30 m , 5.0 nC is enclosed: $E = 8.99 \times 10^9 \times 5.0 \times 10^{-9} / 0.090 = \mathbf{5.0 \times 10^2 \text{ N/C}}$.
3. $\mathbf{E = 0}$ in the metal.

Problem 20 Long charged cylinder

1. Per length L , $Q_{\text{enc}} = \rho \pi r^2 L$, so $E(2\pi r L) = \rho \pi r^2 L/\epsilon_0$ and $E = \rho r/(2\epsilon_0) = 2.0 \times 10^{-6} \times 0.030 / (2 \times 8.85 \times 10^{-12}) = \mathbf{3.4 \times 10^3 \text{ N/C}}$.
2. Outside, all charge within R counts: $E = \rho R^2/(2\epsilon_0 r) = 2.0 \times 10^{-6} \times 0.0025 / (1.77 \times 10^{-11} \times 0.10) = \mathbf{2.8 \times 10^3 \text{ N/C}}$.