

Gravitational force, potential and satellite orbits

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: Newton's law of gravitation says two masses attract with force $F = G m_1 m_2 / r^2$. The field strength of a planet is $g = GM/r^2$, its potential is $V = -GM/r$, and a satellite in circular orbit has speed $v = \sqrt{GM/r}$ and period $T^2 = 4\pi^2 r^3 / (GM)$, which is Kepler's third law.

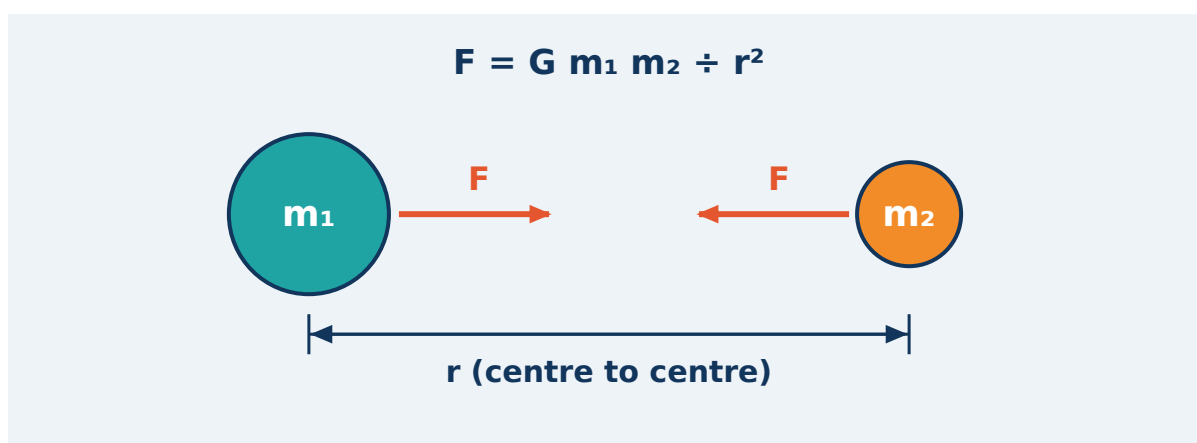
Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB D.1

What does Newton's law of gravitation say?

Every mass attracts every other mass. The force is tiny for everyday objects but huge for planets. Newton showed that it depends on the product of the masses and falls with the square of the distance between their centres.

Key law

Newton's law of gravitation: $F = G m_1 m_2 \div r^2$, where $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. The force acts along the line joining the centres of the masses, and the two forces are equal and opposite.



Newton's law of gravitation: the forces are equal and opposite and fall with the square of the distance.

Worked example 1

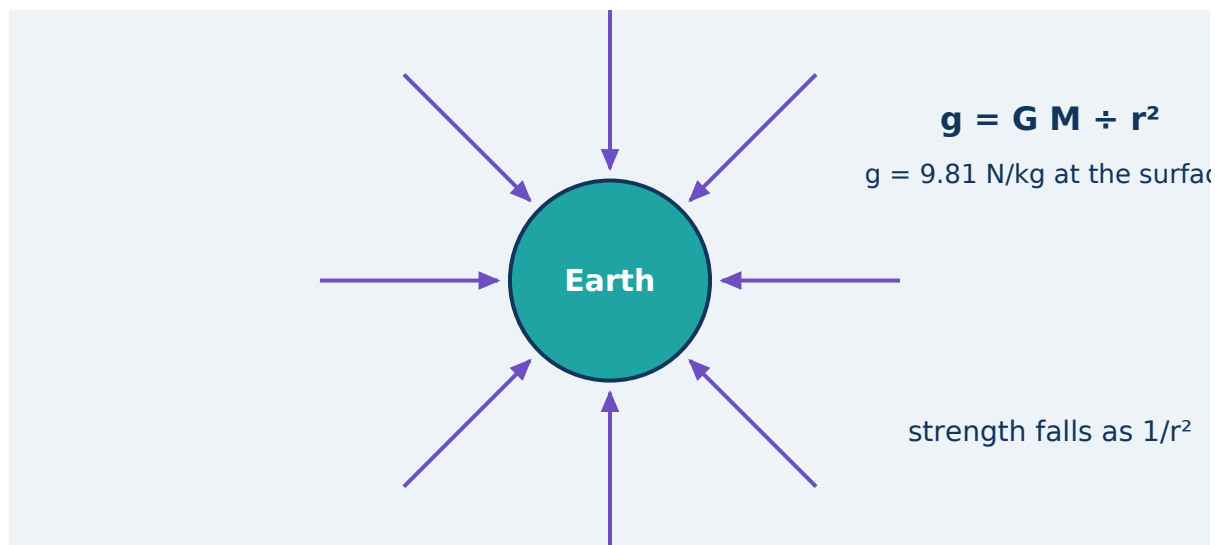
Earth ($5.97 \times 10^{24} \text{ kg}$) and the Moon ($7.35 \times 10^{22} \text{ kg}$) have centres $3.84 \times 10^8 \text{ m}$ apart.
 $F = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22} \div (3.84 \times 10^8)^2 = \mathbf{1.98 \times 10^{20} \text{ N}}$.

What is gravitational field strength?

Definition

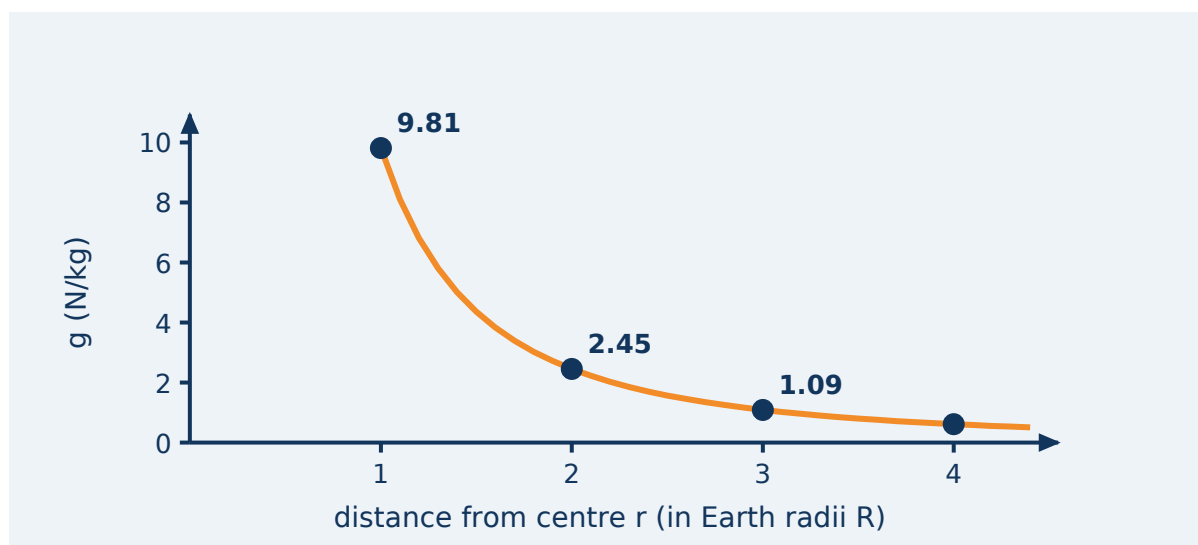
Gravitational field strength g is the gravitational force per unit mass on a small test

mass: $g = F \div m$, in N/kg. It is a vector that points towards the source mass. Outside a spherical body of mass M , $g = G M \div r^2$.



The field of a spherical planet points to its centre. Its strength is $g = G M \div r^2$.

At the Earth's surface $g = G M \div R^2 = 9.81 \text{ N/kg}$, which is the same as 9.81 m/s^2 . The field is not uniform far from the planet. It follows an inverse-square law, as the graph shows.



Field strength falls as $1/r^2$: double the distance and g falls to a quarter.

Worked example 2

The International Space Station orbits 400 km above the surface, so $r = 6.37 \times 10^6 + 4.0 \times 10^5 = 6.77 \times 10^6 \text{ m}$.

$g = G M \div r^2 = 3.98 \times 10^{14} \div (6.77 \times 10^6)^2 = \mathbf{8.69 \text{ N/kg}}$, about 89 % of its surface value. Astronauts float because they and the station are in free fall, not because gravity is missing.

What are gravitational potential and escape speed?

The **gravitational potential** V at a point is the work done per unit mass to bring a small mass

from infinity to that point. It is zero at infinity and negative everywhere nearer, because gravity does positive work as the mass comes in. The potential energy of a mass m is $E_p = m V$.

Formulae

$$V = -G M \div r, E_p = -G M m \div r, v_{\text{esc}} = \sqrt{(2 G M \div R)}$$

V = potential (J/kg), E_p = potential energy (J), M = mass of the planet (kg), m = mass of the object (kg), R = radius of the planet (m), v_{esc} = escape speed (m/s).

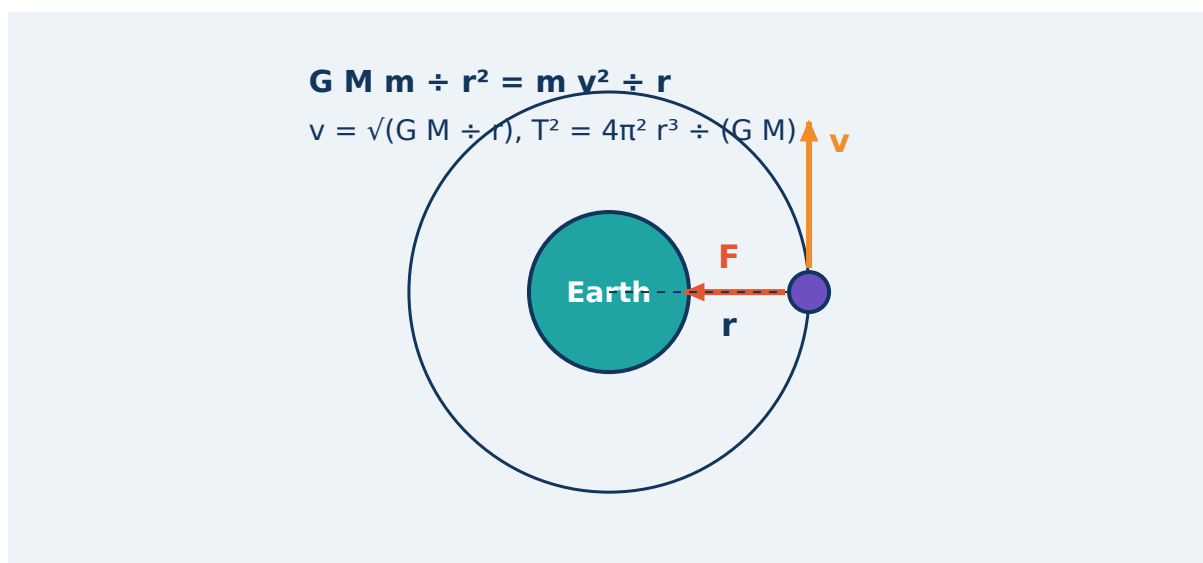
The escape speed is the launch speed for which the kinetic energy exactly cancels the potential energy, $\frac{1}{2} m v^2 = G M m \div R$, so the object just reaches infinity with no speed left.

Worked example 3

Escape speed from the Earth: $v = \sqrt{(2 G M \div R)} = \sqrt{(2 \times 3.98 \times 10^{14} \div 6.37 \times 10^6)} = \sqrt{(1.25 \times 10^8)} = \mathbf{1.12 \times 10^4 \text{ m/s}}$, or 11.2 km/s. The potential at the surface is $V = -3.98 \times 10^{14} \div 6.37 \times 10^6 = -6.25 \times 10^7 \text{ J/kg}$.

How do satellites and Kepler's third law work?

For a circular orbit, gravity provides the centripetal force: $G M m \div r^2 = m v^2 \div r$. This gives $v = \sqrt{(G M \div r)}$ and, since $v = 2\pi r \div T$, $T^2 = 4\pi^2 r^3 \div (G M)$. This is Kepler's third law: T^2 is proportional to r^3 for all bodies orbiting the same mass.



In a circular orbit gravity supplies the centripetal force, which gives the orbital speed and Kepler's third law.

Quantity	Formula	Unit
Force between masses	$G m_1 m_2 \div r^2$	N
Field strength	$G M \div r^2$	N/kg
Potential	$-G M \div r$	J/kg
Orbital speed	$\sqrt{(G M \div r)}$	m/s
Period	$2\pi \sqrt{(r^3 \div G M)}$	s

Worked example 4

A geostationary satellite has a period of $24 \text{ h} = 86\,400 \text{ s}$.

$r^3 = G M T^2 \div 4\pi^2 = 3.98 \times 10^{14} \times 86\,400^2 \div 39.5 = 7.53 \times 10^{22} \text{ m}^3$, so $r = \mathbf{4.22 \times 10^7 \text{ m}}$, which is $3.59 \times 10^7 \text{ m}$ above the surface.

$v = 2\pi r \div T = \mathbf{3.07 \text{ km/s}}$.

Common mistake

Students use the height above the surface as r . In every formula, r is the distance from the *centre* of the planet, so add the planet's radius to the altitude. A faster orbit is a lower orbit, not a higher one.

Lina's tip

Memorise one equation, $G M \div r^2 = v^2 \div r$, and derive the rest. Also remember $G M = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$ for the Earth, so you can skip the multiplication each time.

Key points to remember

- Newton's law: $F = G m_1 m_2 \div r^2$, with $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.
- Field strength $g = F/m = G M \div r^2$, directed towards the source; 9.81 N/kg at Earth's surface.
- Potential $V = -G M \div r$ (zero at infinity); escape speed $v = \sqrt{2 G M \div R}$.
- Circular orbit: $v = \sqrt{G M \div r}$ and $T^2 = 4\pi^2 r^3 \div (G M)$.
- r is always the distance from the centre of the planet, not the height above the surface.

Gravitational force, potential and satellite orbits: frequently asked questions

Why is gravitational potential negative?

Potential is defined as zero at infinity. Gravity is attractive, so it does positive work when a mass moves in from infinity, and the work you must do to take the mass back is positive. Therefore the potential at any finite distance is below zero.

Why do astronauts feel weightless in orbit?

The astronauts and their spacecraft fall freely around the Earth with the same acceleration. There is no contact force between them and the floor, so they feel weightless although gravity is still about 89 % of its surface value.

What is the difference between g and G ?

G is the universal gravitational constant, $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, the same everywhere. g is the field strength at a particular place, which depends on the mass of the planet and the distance, and is 9.81 N/kg at the Earth's surface.

Does the escape speed depend on the mass of the rocket?

No. The kinetic energy and the potential energy are both proportional to the mass of the rocket, so it cancels, and the escape speed depends only on the planet's mass and radius.

What does Kepler's third law say?

The square of the orbital period is proportional to the cube of the orbital radius for bodies orbiting the same central mass. For circular orbits, $T^2 = 4\pi^2 r^3 \div (G M)$.