

# Gravitational force, potential and satellite orbits

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

These twenty problems cover Newton's law of gravitation, field strength, potential, escape speed and circular orbits. You will use the inverse-square law, read a field graph, apply Kepler's third law to planets and moons, and combine forces and energy for satellites and double stars.

They suit A Level, IB physics and CBSE Class 11 students. Use  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$  and  $G M(\text{Earth}) = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$ ;  $r$  is always measured from the centre of the planet.

**Level:** Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB D.1

## Problem 1 Key relationships (easy)

Write each relationship.

1. Newton's law of gravitation.
2. The field strength a distance  $r$  from a spherical mass  $M$ .
3. The gravitational potential at distance  $r$  from  $M$ .
4. The orbital speed and the period for a circular orbit.

## Problem 2 Two people close together (easy)

Two students of mass 70 kg stand 1.0 m apart. Use  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ .

1. Calculate the gravitational force between them.
2. Compare it with the weight of one student (use  $g = 9.81 \text{ N/kg}$ ).

## Problem 3 Surface gravity of the Earth (easy)

The Earth has mass  $5.97 \times 10^{24} \text{ kg}$  and radius  $6.37 \times 10^6 \text{ m}$ .

1. Calculate the field strength at the surface.
2. Explain why the answer is the same as the free-fall acceleration.

## Problem 4 Weight on the Moon (easy)

The Moon has mass  $7.35 \times 10^{22} \text{ kg}$  and radius  $1.74 \times 10^6 \text{ m}$ . An astronaut has a mass of 60 kg.

1. Calculate the field strength at the surface of the Moon.
2. Calculate the astronaut's weight on the Moon and on the Earth.
3. State the astronaut's mass on the Moon.

### Problem 5 Inverse-square thinking (easy)

The gravitational force between two masses is  $F$ . State the new force when:

1. the distance is tripled;
2. one mass is doubled;
3. both masses are doubled and the distance is doubled;
4. the distance is halved.

### Problem 6 Potential at the surface (easy)

Use  $G M = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$  and  $R = 6.37 \times 10^6 \text{ m}$  for the Earth.

1. Calculate the potential at the Earth's surface.
2. State what the negative sign means.
3. Calculate the energy needed to take a  $2.0 \text{ kg}$  mass from the surface to infinity.

### Problem 7 Moons of Jupiter (medium)

Io orbits Jupiter in a nearly circular orbit of radius  $4.22 \times 10^8 \text{ m}$  with a period of 1.77 days.

1. Convert the period to seconds.
2. Use  $T^2 = 4\pi^2 r^3 \div (G M)$  to find the mass of Jupiter.

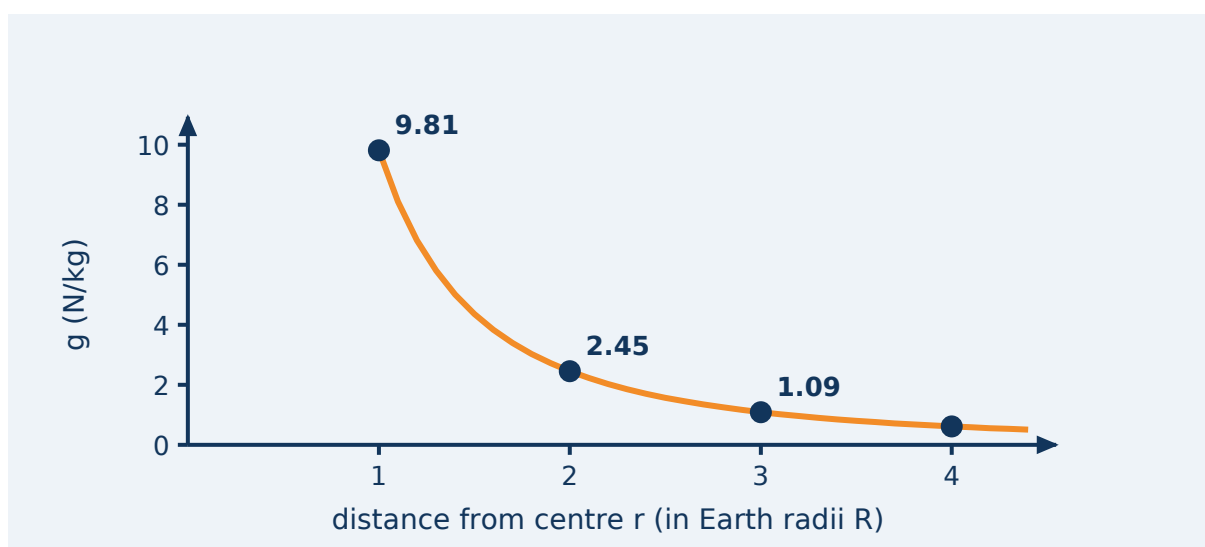
### Problem 8 Half the surface gravity (medium)

Take the Earth's radius as  $R = 6.37 \times 10^6 \text{ m}$ .

1. Show that  $g$  is half its surface value when  $r = \sqrt{2} R$ .
2. Calculate the height above the surface at which  $g = 4.9 \text{ N/kg}$ .

### Problem 9 Reading a $g$ - $r$ graph (medium)

The graph shows how the Earth's field strength varies with distance from its centre.



Field strength of the Earth against distance from its centre.

1. Read the value of  $g$  at  $r = 2R$ .

2. Show that  $g r^2$  is constant for the points at R, 2R and 3R.
3. Calculate the change in potential between R and 3R.

### Problem 10 Orbit of the space station (medium)

A space station orbits 410 km above the Earth's surface. Use  $G M = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$  and  $R = 6.37 \times 10^6 \text{ m}$ .

1. Calculate the orbital radius.
2. Calculate the orbital speed.
3. Calculate the period in minutes.

### Problem 11 Kepler on the planets (medium)

Use  $T^2 \propto r^3$  with T in years and r in astronomical units (AU), so that the Earth has  $T = 1$  year and  $r = 1$  AU.

1. Mars orbits at 1.52 AU. Calculate its period.
2. A comet has a period of 76 years. Calculate the mean orbital radius (the semi-major axis) in AU.

### Problem 12 Weighing the Sun (medium)

The Earth orbits the Sun in an almost circular orbit of radius  $1.50 \times 10^{11} \text{ m}$  with a period of  $3.156 \times 10^7 \text{ s}$  (one year).

1. Show that the mass of the Sun is  $M = 4\pi^2 r^3 \div (G T^2)$ .
2. Calculate the mass of the Sun.
3. Compare it with the Earth's mass of  $5.97 \times 10^{24} \text{ kg}$ .

### Problem 13 Leaving the Earth and Moon (medium)

Use  $G M = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$  and  $R = 6.37 \times 10^6 \text{ m}$  for the Earth; the Moon has  $G M = 4.90 \times 10^{12} \text{ m}^3/\text{s}^2$  and  $R = 1.74 \times 10^6 \text{ m}$ .

1. Calculate the escape speed from the Earth.
2. Calculate the escape speed from the Moon.
3. Calculate the kinetic energy a 500 kg probe needs at the Earth's surface to escape.

### Problem 14 Sun against Moon (medium)

The Sun has mass  $1.99 \times 10^{30} \text{ kg}$  and is  $1.50 \times 10^{11} \text{ m}$  from the Earth. The Moon has mass  $7.35 \times 10^{22} \text{ kg}$  and is  $3.84 \times 10^8 \text{ m}$  from the Earth.

1. Calculate the ratio of the Sun's pull on the Earth to the Moon's pull.
2. Suggest why the Moon still causes larger tides than the Sun.

### Problem 15 No gravity in orbit? (medium)

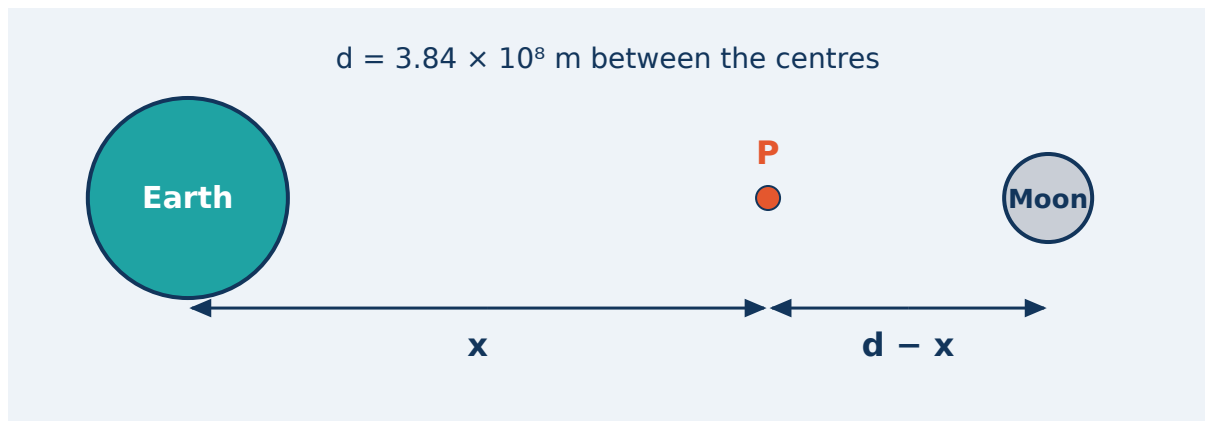
A student says: "Astronauts on the space station float because there is no gravity at 400 km

up."

1. Calculate the field strength at 400 km above the surface ( $r = 6.77 \times 10^6$  m).
2. Express it as a percentage of 9.81 N/kg.
3. Explain correctly why astronauts appear weightless.

### Problem 16 The neutral point (hard)

At a point P on the line between the Earth and the Moon the fields cancel. Use  $M_E = 5.97 \times 10^{24}$  kg,  $M_M = 7.35 \times 10^{22}$  kg and  $d = 3.84 \times 10^8$  m.



*The neutral point lies on the line between the Earth and the Moon.*

1. Write the condition for the fields to cancel in terms of  $x$  and  $d - x$ .
2. Show that  $x \div (d - x) = \sqrt{(M_E \div M_M)}$ .
3. Calculate  $x$ .

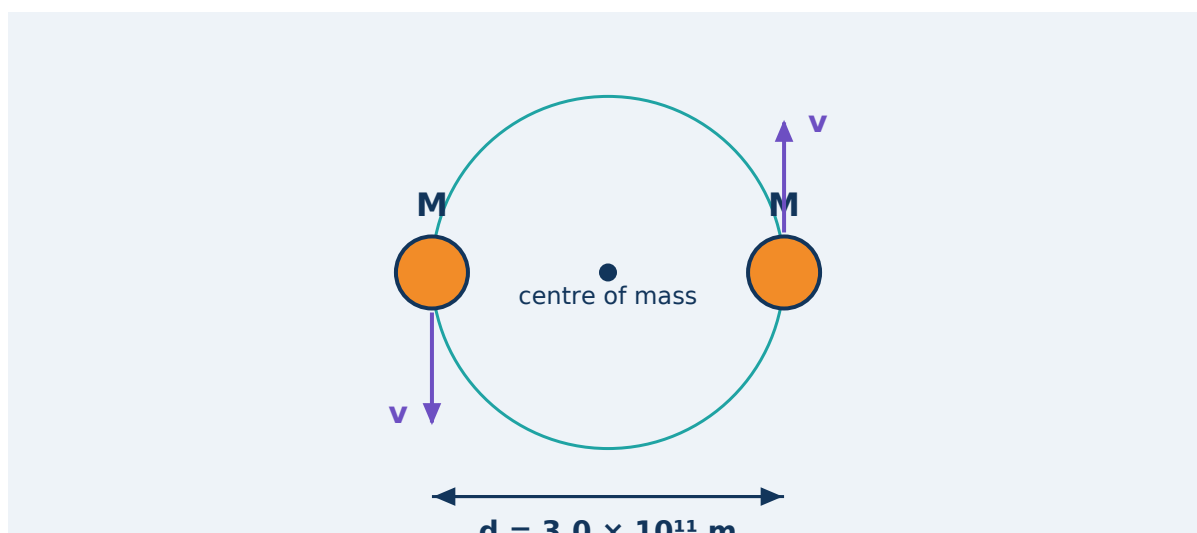
### Problem 17 Energy to reach orbit (hard)

A satellite of mass 1200 kg is lifted from the surface of the Earth to a circular orbit of radius  $8.0 \times 10^6$  m. Ignore the rotation of the Earth and use  $G M = 3.98 \times 10^{14}$  m<sup>3</sup>/s<sup>2</sup>,  $R = 6.37 \times 10^6$  m.

1. Calculate the kinetic energy in the orbit.
2. Calculate the total energy in the orbit.
3. Calculate the total energy on the ground.
4. Calculate the minimum energy that must be supplied.

### Problem 18 Binary stars (hard)

Two stars, each of mass  $2.0 \times 10^{30}$  kg, orbit their common centre with a separation of  $3.0 \times 10^{11}$  m.



Two equal stars orbit their common centre.

1. Explain why each star moves on a circle of radius  $d \div 2$ .
2. Show that  $\omega^2 = 2 G M \div d^3$ .
3. Calculate the period in years.
4. Calculate the orbital speed of each star.

### Problem 19 Orbit decay paradox (hard)

A satellite of mass 500 kg orbits at  $r_1 = 6.78 \times 10^6 \text{ m}$ . Atmospheric drag lowers it to a new circular orbit of radius  $r_2 = 6.68 \times 10^6 \text{ m}$ . Use  $G M = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$ .

1. Calculate the orbital speed at  $r_1$  and at  $r_2$ .
2. Calculate the total energy at each radius.
3. Explain why the satellite speeds up although drag acts against its motion.

### Problem 20 Atmosphere of the Moon (hard)

The root-mean-square speed of gas molecules is  $v = \sqrt{3 k T \div m}$ , with  $k = 1.38 \times 10^{-23} \text{ J/K}$ . At 300 K a nitrogen molecule has  $m = 4.65 \times 10^{-26} \text{ kg}$  and a hydrogen molecule has  $m = 3.34 \times 10^{-27} \text{ kg}$ . A gas escapes over geological time if its mean speed exceeds about one sixth of the escape speed.

1. Calculate the r.m.s. speed of nitrogen and of hydrogen at 300 K.
2. Calculate one sixth of the escape speed for the Earth (11.2 km/s) and for the Moon (2.37 km/s).
3. Predict which gases the Earth and the Moon retain.