

Gravitational force, potential and satellite orbits

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer states the law used, then the substitution and the result with its unit. Check that you measured r from the centre of the planet, converted kilometres to metres and days to seconds, and kept enough digits until the last step.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB D.1

Problem 1 Key relationships

- $F = G m_1 m_2 \div r^2$.
 - $g = G M \div r^2$.
 - $V = -G M \div r$.
 - $v = \sqrt{G M \div r}$ and $T^2 = 4\pi^2 r^3 \div (G M)$.
- $F = Gm_1m_2/r^2$; $g = GM/r^2$; $V = -GM/r$; $v = \sqrt{GM/r}$.**

Problem 2 Two people close together

- $F = G m_1 m_2 \div r^2 = 6.67 \times 10^{-11} \times 70 \times 70 \div 1.0^2 = 3.27 \times 10^{-7} \text{ N}$.
 - Weight = $70 \times 9.81 = 687 \text{ N}$, so the force is about 5×10^{-10} of the weight, which is negligible.
- $F = 3.3 \times 10^{-7} \text{ N}$.**

Problem 3 Surface gravity of the Earth

- $g = G M \div R^2 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \div (6.37 \times 10^6)^2 = 3.98 \times 10^{14} \div 4.06 \times 10^{13} = 9.81 \text{ N/kg}$.
 - $g = F \div m$ and $F = m a$ for a freely falling mass, so $a = g$. Hence N/kg and m/s^2 are equivalent.
- $g = 9.81 \text{ N/kg}$.**

Problem 4 Weight on the Moon

- $g = 6.67 \times 10^{-11} \times 7.35 \times 10^{22} \div (1.74 \times 10^6)^2 = 4.90 \times 10^{12} \div 3.03 \times 10^{12} = 1.62 \text{ N/kg}$.
 - Moon: $60 \times 1.62 = 97 \text{ N}$. Earth: $60 \times 9.81 = 589 \text{ N}$.
 - The mass is unchanged at 60 kg .
- $g = 1.62 \text{ N/kg}$; 97 N ; 589 N ; 60 kg .**

Problem 5 Inverse-square thinking

1. $F \propto 1/r^2$, so the force is $F \div 9$.
2. $F \propto m_1$, so the force is $2F$.
3. $2 \times 2 \div 2^2 = 1$, so the force stays F .
4. $1 \div (1/2)^2 = 4$, so the force is $4F$.

F/9; 2F; F; 4F.

Problem 6 Potential at the surface

1. $V = -G M \div R = -3.98 \times 10^{14} \div 6.37 \times 10^6 = -6.25 \times 10^7 \text{ J/kg}$.
2. The potential is lower than at infinity, so work must be done on the mass to remove it.
3. $\Delta E = m \times (0 - V) = 2.0 \times 6.25 \times 10^7 = 1.25 \times 10^8 \text{ J}$.

V = -6.25 × 10⁷ J/kg; 1.25 × 10⁸ J.

Problem 7 Moons of Jupiter

1. $T = 1.77 \times 86\,400 = 1.53 \times 10^5 \text{ s}$.
2. $M = 4\pi^2 r^3 \div (G T^2) = 39.48 \times (4.22 \times 10^8)^3 \div (6.67 \times 10^{-11} \times (1.529 \times 10^5)^2) = 2.97 \times 10^{27} \div 1.56 = 1.90 \times 10^{27} \text{ kg}$.

M = 1.9 × 10²⁷ kg.

Problem 8 Half the surface gravity

1. $g \propto 1/r^2$, so $g \div g_0 = R^2 \div r^2 = 1/2$ gives $r^2 = 2R^2$ and $r = \sqrt{2} R$.
2. $h = r - R = (\sqrt{2} - 1) \times 6.37 \times 10^6 = 0.414 \times 6.37 \times 10^6 = 2.64 \times 10^6 \text{ m}$.

h = 2.64 × 10⁶ m.

Problem 9 Reading a g-r graph

1. At $r = 2R$, $g = 2.45 \text{ N/kg}$.
2. In units of R^2 , $g r^2 = 9.81 \times 1 = 9.81$; $2.45 \times 4 = 9.80$; $1.09 \times 9 = 9.81$. The product is constant, so $g \propto 1/r^2$.
3. $\Delta V = G M (1/R - 1/3R) = 6.25 \times 10^7 \times (2/3) = 4.17 \times 10^7 \text{ J/kg}$, which equals the area under the graph between R and $3R$.

ΔV = 4.17 × 10⁷ J/kg.

Problem 10 Orbit of the space station

1. $r = 6.37 \times 10^6 + 4.1 \times 10^5 = 6.78 \times 10^6 \text{ m}$.
2. $v = \sqrt{G M \div r} = \sqrt{3.98 \times 10^{14} \div 6.78 \times 10^6} = \sqrt{5.87 \times 10^7} = 7.66 \times 10^3 \text{ m/s}$.
3. $T = 2\pi r \div v = 2\pi \times 6.78 \times 10^6 \div 7663 = 5560 \text{ s} = 92.7 \text{ min}$.

v = 7.66 km/s; T = 92.7 min.

Problem 11 Kepler on the planets

1. $T = r^{1.5} = 1.52 \times \sqrt{1.52} = 1.52 \times 1.233 = 1.87$ years.

2. $r^3 = T^2 = 5776$, so $r = 5776^{(1/3)} = 17.9$ AU.

T = 1.87 years; r = 17.9 AU.

Problem 12 Weighing the Sun

1. The Sun's pull supplies the centripetal force: $G M m \div r^2 = m (4\pi^2 r \div T^2)$, so $M = 4\pi^2 r^3 \div (G T^2)$.

2. $M = 39.48 \times (1.50 \times 10^{11})^3 \div (6.67 \times 10^{-11} \times (3.156 \times 10^7)^2) = 1.332 \times 10^{35} \div 6.64 \times 10^4 = 2.01 \times 10^{30}$ kg.

3. $2.01 \times 10^{30} \div 5.97 \times 10^{24} = 3.4 \times 10^5$, so the Sun is about 340 000 times as massive as the Earth.

M = 2.01 × 10³⁰ kg.

Problem 13 Leaving the Earth and Moon

1. $v = \sqrt{2 G M \div R} = \sqrt{2 \times 3.98 \times 10^{14} \div 6.37 \times 10^6} = 1.12 \times 10^4$ m/s.

2. $v = \sqrt{2 \times 4.90 \times 10^{12} \div 1.74 \times 10^6} = \sqrt{5.63 \times 10^6} = 2.37 \times 10^3$ m/s.

3. $KE = G M m \div R = 6.25 \times 10^7 \times 500 = 3.13 \times 10^{10}$ J.

11.2 km/s; 2.37 km/s; 3.13 × 10¹⁰ J.

Problem 14 Sun against Moon

1. Ratio = $(M_S \div M_M) \times (d_M \div d_S)^2 = 2.71 \times 10^7 \times (2.56 \times 10^{-3})^2 = 177$.

2. Tides depend on the *difference* in the pull across the Earth, which falls as $1/d^3$. The Moon is much closer, so its difference is larger.

Sun's pull is about 177 times larger.

Problem 15 No gravity in orbit?

1. $g = 3.98 \times 10^{14} \div (6.77 \times 10^6)^2 = 8.69$ N/kg.

2. $8.69 \div 9.81 \times 100 \% = 88.6 \%$.

3. The station and astronauts fall together with the same acceleration, so there is no contact force between them and they feel weightless.

g = 8.69 N/kg (89 %).

Problem 16 The neutral point

1. $G M_E \div x^2 = G M_M \div (d - x)^2$.

2. Rearranging, $(d - x)^2 \div x^2 = M_M \div M_E$, so $x \div (d - x) = \sqrt{M_E \div M_M} = \sqrt{81.2} = 9.01$.

3. $x = 9.01 (d - x)$ gives $x = 9.01 d \div 10.01 = 0.900 \times 3.84 \times 10^8 = 3.46 \times 10^8$ m from the Earth's centre.

x = 3.46 × 10⁸ m.

Problem 17 Energy to reach orbit

1. $KE = G M m \div 2r = 3.98 \times 10^{14} \times 1200 \div (1.6 \times 10^7) = 2.99 \times 10^{10} \text{ J}.$
2. $E_{\text{total}} = -G M m \div 2r = -2.99 \times 10^{10} \text{ J}.$
3. At rest on the ground $E = -G M m \div R = -3.98 \times 10^{14} \times 1200 \div 6.37 \times 10^6 = -7.50 \times 10^{10} \text{ J}.$
4. Energy supplied $= -2.99 \times 10^{10} - (-7.50 \times 10^{10}) = 4.51 \times 10^{10} \text{ J}.$
KE = $2.99 \times 10^{10} \text{ J}$; energy supplied = $4.51 \times 10^{10} \text{ J}$.

Problem 18 Binary stars

1. The stars have equal masses, so the centre of mass is at the midpoint and both circle it with the same ω .
2. $G M^2 \div d^2 = M \omega^2 (d \div 2)$, so $\omega^2 = 2 G M \div d^3$.
3. $\omega^2 = 2 \times 6.67 \times 10^{-11} \times 2.0 \times 10^{30} \div (3.0 \times 10^{11})^3 = 9.88 \times 10^{-15}$, so $\omega = 9.94 \times 10^{-8} \text{ rad/s}$ and $T = 2\pi \div \omega = 6.32 \times 10^7 \text{ s} = 2.0 \text{ years}.$
4. $v = \omega \times d \div 2 = 9.94 \times 10^{-8} \times 1.5 \times 10^{11} = 1.49 \times 10^4 \text{ m/s}.$
T = 2.0 years; v = 14.9 km/s.

Problem 19 Orbit decay paradox

1. $v_1 = \sqrt{(3.98 \times 10^{14} \div 6.78 \times 10^6)} = 7664 \text{ m/s}$ and $v_2 = \sqrt{(3.98 \times 10^{14} \div 6.68 \times 10^6)} = 7720 \text{ m/s}.$
2. $E = -G M m \div 2r$: $E_1 = -1.468 \times 10^{10} \text{ J}$ and $E_2 = -1.490 \times 10^{10} \text{ J}$. The total energy fell by $2.2 \times 10^8 \text{ J}.$
3. Gravitational potential energy falls by twice as much ($4.4 \times 10^8 \text{ J}$), so half of it becomes extra kinetic energy ($2.2 \times 10^8 \text{ J}$) and the other half is the energy dissipated by drag. The kinetic energy rises, so the speed rises.
 $v_1 = 7.66 \text{ km/s}$; $v_2 = 7.72 \text{ km/s}$; total energy lost $2.2 \times 10^8 \text{ J}$.

Problem 20 Atmosphere of the Moon

1. N_2 : $v = \sqrt{(3 \times 1.38 \times 10^{-23} \times 300 \div 4.65 \times 10^{-26})} = \sqrt{(2.67 \times 10^5)} = 517 \text{ m/s}$. H_2 : $v = \sqrt{(3 \times 1.38 \times 10^{-23} \times 300 \div 3.34 \times 10^{-27})} = 1930 \text{ m/s}.$
2. Earth: $11\,200 \div 6 = 1870 \text{ m/s}$. Moon: $2370 \div 6 = 395 \text{ m/s}.$
3. Earth: nitrogen (517 m/s) is below 1870 m/s so it stays; hydrogen (1930 m/s) is above it and escapes. Moon: both are above 395 m/s, so the Moon cannot keep an atmosphere.
517 m/s and 1930 m/s; Earth keeps nitrogen but loses hydrogen; the Moon loses both.