

Gas laws, kinetic theory and thermal energy

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: An ideal gas obeys $pV = nRT$, where pressure, volume and kelvin temperature are linked through the amount of gas. Kinetic theory explains this: the mean kinetic energy of a molecule is $(3/2)kT$, and the internal energy of a monatomic ideal gas is $(3/2)nRT$. Added heat changes temperature by $Q = mc\Delta T$ or the state by $Q = mL$.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB B.1-B.3

What makes a gas ideal?

An **ideal gas** is a model that lets physicists predict how the pressure, volume and temperature of a gas change together. It works well for air, helium or nitrogen at room conditions, and it fails only when a gas is squashed hard or cooled close to its boiling point.

Definition

An **ideal gas** is a gas whose particles are point-like, never attract or repel each other except during collisions, and collide elastically with the walls and with each other.

These assumptions matter because they remove any potential energy between particles. The only energy stored in the gas is the random kinetic energy of its particles, so the **internal energy** depends on temperature alone.

How do you use $pV = nRT$?

The equation of state links the four measurable quantities of a gas. A sealed tyre, a diver's cylinder and a balloon all obey it to a good approximation.

Formula

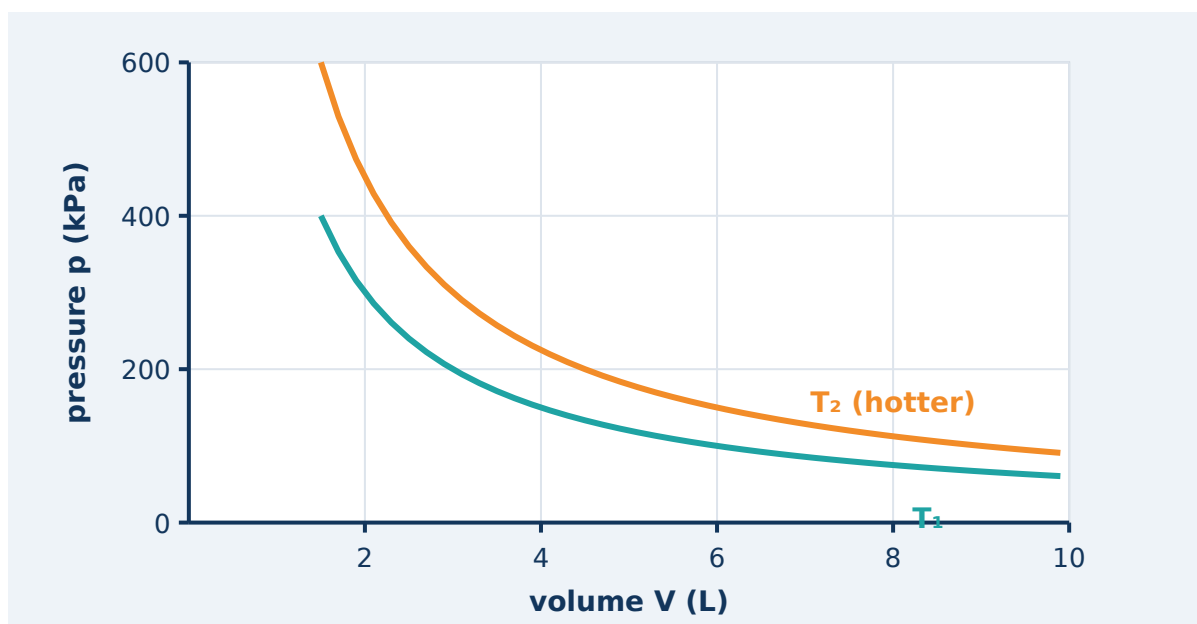
$pV = nRT$ (also written **$pV = NkT$**)

p = pressure (Pa), V = volume (m^3), n = amount of gas (mol), N = number of molecules, T = temperature (K), $R = 8.31 \text{ J}/(\text{mol K})$, $k = 1.38 \times 10^{-23} \text{ J/K}$, and $R = N_A k$ with $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$.

Worked example

A 5.0 L cylinder holds 0.40 mol of gas at 27 °C. Calculate the pressure.

Convert first: $T = 27 + 273 = 300 \text{ K}$ and $V = 5.0 \text{ L} = 5.0 \times 10^{-3} \text{ m}^3$. Then $p = nRT / V = (0.40 \times 8.31 \times 300) / (5.0 \times 10^{-3}) = 1.99 \times 10^5 \text{ Pa}$, so **$p \approx 2.0 \times 10^5 \text{ Pa}$** .



Isotherms of a fixed amount of ideal gas: at a higher temperature the curve sits higher, but $p \times V$ is still constant along each curve.

Common mistake

Never put a Celsius temperature or a volume in litres into $pV = nRT$. The answer is wrong by a factor of 300 or 1000 and still looks like a sensible number.

How are temperature and molecular motion linked?

Kinetic theory connects the large-scale temperature to the motion of individual molecules. The hotter the gas, the faster its molecules move on average.

Key law

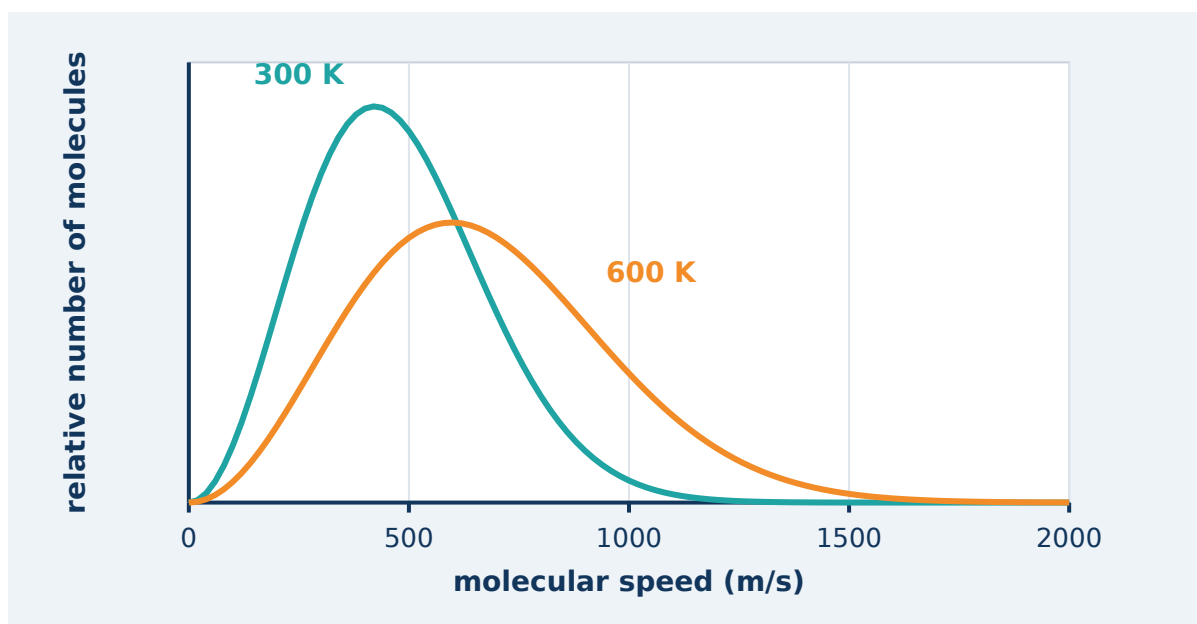
The mean kinetic energy of a molecule is $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$. For n moles of a monatomic ideal gas the internal energy is $U = \frac{3}{2}nRT$. The root-mean-square speed is $c_{rms} = \sqrt{3RT / M}$, with M the molar mass in kg/mol.

Worked example

Find the r.m.s. speed of nitrogen molecules ($M = 0.028$ kg/mol) at 300 K and their mean kinetic energy.

$$c_{rms} = \sqrt{3 \times 8.31 \times 300 / 0.028} = \sqrt{2.67 \times 10^5} = \mathbf{517 \text{ m/s}}$$

Mean kinetic energy = $\frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = \mathbf{6.21 \times 10^{-21} \text{ J}}$.



Speed distribution of nitrogen molecules. Heating moves the peak to higher speed, flattens it and widens it.

Notice that doubling the kelvin temperature doubles the mean kinetic energy but multiplies the r.m.s. speed by only $\sqrt{2}$, about 1.41.

How much energy changes the temperature or the state?

Thermal energy added to a solid or liquid raises its temperature or changes its state, never both at once. While a substance melts or boils, the energy goes into breaking bonds, so the temperature stays flat.

Formula

$$Q = mc\Delta T \text{ and } Q = mL$$

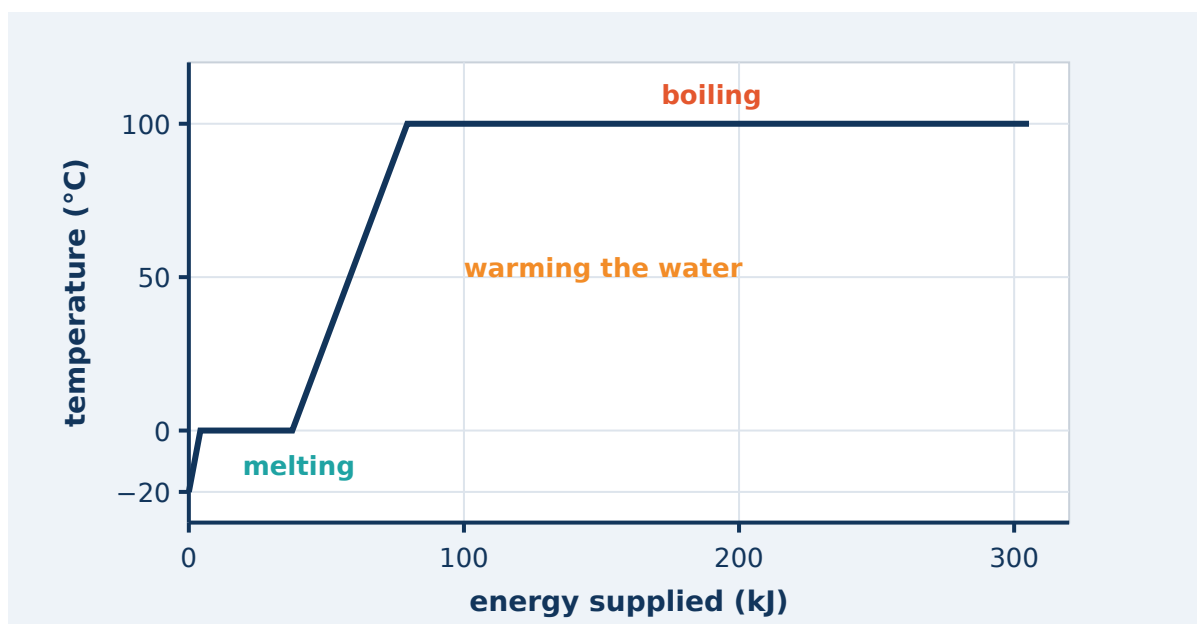
Q = energy transferred (J), m = mass (kg), c = specific heat capacity (J/(kg K)), ΔT = temperature change (K or °C), L = specific latent heat (J/kg).

Substance	Value	Meaning
Water (liquid)	4180 J/(kg K)	specific heat capacity
Ice	2100 J/(kg K)	specific heat capacity
Aluminium	900 J/(kg K)	specific heat capacity
Water, melting	3.34×10^5 J/kg	latent heat of fusion
Water, boiling	2.26×10^6 J/kg	latent heat of vaporisation

Worked example

How much energy turns 0.10 kg of ice at -20°C into water at 0°C ?

Warming the ice: $Q_1 = 0.10 \times 2100 \times 20 = 4200$ J. Melting it: $Q_2 = 0.10 \times 3.34 \times 10^5 = 33\,400$ J. Total = $4200 + 33\,400 = \mathbf{37\,600\,J}$, matching the curve below.



Heating curve for 0.10 kg of ice. The long plateau at 100 °C is the latent heat of vaporisation.

Lina's tip

Write the plan before you calculate: warm, melt, warm, boil. Each stage has its own formula and the stages simply add.

Boiling needs far more energy than melting because the molecules must be separated completely and must push back the atmosphere. For 0.10 kg of water the vaporisation stage costs 226 000 J, nearly seven times the melting stage.

Key points to remember

- $pV = nRT$ with p in Pa, V in m^3 , T in K and $R = 8.31 \text{ J/(mol K)}$
- Mean kinetic energy of a molecule = $(3/2)kT$; internal energy $U = (3/2)nRT$
- r.m.s. speed $c = \sqrt{3RT / M}$, with M in kg/mol
- $Q = mc\Delta T$ changes temperature; $Q = mL$ changes state at constant temperature

Gas laws, kinetic theory and thermal energy: frequently asked questions

What is an ideal gas in physics?

It is a model gas whose particles have negligible volume and exert no forces on each other except in collisions. Real gases such as air behave almost ideally at normal pressure and temperature.

Why must temperature be in kelvin in $pV = nRT$?

The equation is a proportionality between pV and absolute temperature, which starts from zero when molecular motion is minimal. Celsius values can be zero or negative while the gas still has pressure, so they give wrong results.

What is the internal energy of an ideal gas?

It is the total random kinetic energy of all its particles, since there is no potential energy between them. For a monatomic gas $U = (3/2)nRT$, so it depends only on temperature.

What is the difference between specific heat capacity and latent heat?

Specific heat capacity is the energy needed to raise 1 kg of a substance by 1 K without changing its state. Latent heat is the energy needed to change the state of 1 kg at constant temperature.

Why does the temperature stay constant during melting?

The energy supplied increases the potential energy of the particles as bonds are broken, not their kinetic energy. Temperature measures mean kinetic energy, so it does not change until all the solid has melted.