

Gas laws, kinetic theory and thermal energy

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit. Check that you converted to kelvin, cubic metres and kilograms before you compare your numbers, and note that small rounding differences in the last digit are acceptable.

Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB B.1-B.3

Problem 1 Kelvin conversions

$$T = \theta + 273.$$

1. $37 + 273 = \mathbf{310\ K}.$

2. $85 - 273 = \mathbf{-188\ ^\circ C}.$

3. A change of $1\ ^\circ\text{C}$ equals a change of $1\ \text{K}$, so the rise is $\mathbf{25\ K}.$

Problem 2 Moles and molecules

1. $n = m / M = 3.2 / 32 = \mathbf{0.10\ mol}.$

2. $N = nN_A = 0.10 \times 6.02 \times 10^{23} = \mathbf{6.0 \times 10^{22}\ molecules}.$

3. $m = nM = 0.75 \times 44 = \mathbf{33\ g}.$

Problem 3 Pressure from the equation

$$V = 2.0 \times 10^{-3}\ \text{m}^3 \text{ and } T = 300\ \text{K}.$$

$$p = nRT / V = (0.080 \times 8.31 \times 300) / (2.0 \times 10^{-3}) = 99720\ \text{Pa}, \text{ so } \mathbf{p \approx 1.0 \times 10^5\ Pa}.$$

Problem 4 Boiling water for tea

$$Q = mc\Delta T = 2.0 \times 4180 \times (100 - 20) = 668800\ \text{J}, \text{ so } \mathbf{Q \approx 6.7 \times 10^5\ J}.$$

Problem 5 Cool box ice

$$Q = mL = 0.60 \times 3.34 \times 10^5 = 200400\ \text{J}, \text{ so } \mathbf{Q \approx 2.0 \times 10^5\ J}.$$
 The ice stays at $0\ ^\circ\text{C}$ throughout.

Problem 6 Ideas behind the model

- Any three of: the molecules have negligible volume; there are no forces between them except in collisions; collisions are elastic; their motion is random; the number of molecules is very large.
- With no forces between molecules there is no potential energy. The internal energy is therefore just the sum of the random kinetic energies, which is set by the temperature. **U**

depends on T only.

Problem 7 Diver's cylinder

$$T = 293 \text{ K}, V = 0.010 \text{ m}^3.$$

$$1. n = pV / RT = (2.0 \times 10^6 \times 0.010) / (8.31 \times 293) = 8.21 \text{ mol, so } n \approx \mathbf{8.2 \text{ mol}}.$$

$$2. m = nM = 8.214 \times 28 = 230 \text{ g, so } m \approx \mathbf{2.3 \times 10^2 \text{ g}}.$$

Problem 8 Helium at the fair

$$T = 300 \text{ K}.$$

$$1. E = (3/2)kT = 1.5 \times 1.38 \times 10^{-23} \times 300 = \mathbf{6.21 \times 10^{-21} \text{ J}}.$$

$$2. U = (3/2)nRT = 1.5 \times 0.50 \times 8.31 \times 300 = 1870 \text{ J, so } U \approx \mathbf{1.9 \times 10^3 \text{ J}}.$$

Problem 9 Light atoms move fast

$$1. c = \sqrt{3RT / M} = \sqrt{3 \times 8.31 \times 300 / 0.0040} = \sqrt{1869750} = \mathbf{1367 \text{ m/s}}.$$

$$2. \text{At equal temperature } c \propto 1/\sqrt{M}, \text{ so the ratio is } \sqrt{(0.040 / 0.0040)} = \sqrt{10} = \mathbf{3.2}.$$

Problem 10 Motorbike tyre

$$p/T \text{ is constant, with } T_1 = 298 \text{ K and } T_2 = 328 \text{ K}.$$

$$p_2 = p_1 T_2 / T_1 = 2.20 \times 10^5 \times 328 / 298 = 242148 \text{ Pa, so } p_2 \approx \mathbf{2.42 \times 10^5 \text{ Pa}}.$$

Problem 11 Metal block heater

$$E = Pt = 60 \times 300 = 18000 \text{ J}.$$

$$c = E / (m\Delta T) = 18000 / (0.50 \times 40) = \mathbf{900 \text{ J/(kg K)}}, \text{ which matches aluminium}.$$

Problem 12 Tea and milk

$$\text{Energy lost by tea} = \text{energy gained by milk: } 0.20c(90 - T) = 0.050c(T - 5).$$

$$18 - 0.20T = 0.050T - 0.25, \text{ so } 0.25T = 18.25 \text{ and } T = \mathbf{73 \text{ }^\circ\text{C}}.$$

Problem 13 Reading a pressure-volume set

$$1. pV = 100 \text{ kPa} \times 6.0 \text{ L} = 600; 150 \times 4.0 = 600; 200 \times 3.0 = 600; 300 \times 2.0 = 600 \text{ (kPa L, i.e. J). The product is constant, so } T \text{ is constant}.$$

$$2. n = pV / RT = 600 / (8.31 \times 290) = 0.249 \text{ mol, so } n \approx \mathbf{0.25 \text{ mol}}.$$

$$3. p = 600 / 2.4 = \mathbf{250 \text{ kPa}}.$$

Problem 14 Spot the error

$$1. \text{The temperature was left in } ^\circ\text{C} \text{ (it must be } 300 \text{ K)} \text{ and the volume was left in litres (it must be } 4.0 \times 10^{-3} \text{ m}^3).$$

2. $p = (0.20 \times 8.31 \times 300) / (4.0 \times 10^{-3}) = 124650 \text{ Pa}$, so $p \approx \mathbf{1.2 \times 10^5 \text{ Pa}}$.

Problem 15 Air in a classroom

1. $\rho = m/V = pM / RT = (1.01 \times 10^5 \times 0.0290) / (8.31 \times 293) = 1.203 \text{ kg/m}^3$, so $\rho \approx \mathbf{1.20 \text{ kg/m}^3}$.

2. $V = 8.0 \times 6.0 \times 3.0 = 144 \text{ m}^3$, so $m = \rho V = 1.2030 \times 144 = 173.2 \text{ kg}$, so $m \approx \mathbf{1.7 \times 10^2 \text{ kg}}$.

Problem 16 Two flasks, two temperatures

$n_A = pV / RT = (2.0 \times 10^5 \times 3.0 \times 10^{-3}) / (8.31 \times 300) = 0.2407 \text{ mol}$.

$n_B = (1.0 \times 10^5 \times 5.0 \times 10^{-3}) / (8.31 \times 400) = 0.1504 \text{ mol}$.

Total $n = 0.3911 \text{ mol}$. $p = nRT / V = (0.3911 \times 8.31 \times 350) / (8.0 \times 10^{-3}) = 142188 \text{ Pa}$, so $p \approx \mathbf{1.4 \times 10^5 \text{ Pa}}$.

Problem 17 Piston lifting a load

1. $p = p_0 + mg/A = 1.01 \times 10^5 + (4.0 \times 9.81) / (5.0 \times 10^{-3}) = 108848 \text{ Pa} \approx \mathbf{1.09 \times 10^5 \text{ Pa}}$.

2. $\Delta V = nR\Delta T / p = (0.020 \times 8.31 \times 60) / 108848 = 9.16 \times 10^{-5} \text{ m}^3$. Rise = $\Delta V / A = 1.83 \text{ cm}$, so $\approx \mathbf{1.8 \text{ cm}}$.

3. $W = p\Delta V = nR\Delta T = 0.020 \times 8.31 \times 60 = 9.97 \text{ J}$, so $W \approx \mathbf{10 \text{ J}}$.

Problem 18 Heating gas against water

1. $\Delta U = (3/2)nR\Delta T = 1.5 \times 1.0 \times 8.31 \times 100 = \mathbf{1246.5 \text{ J}}$ ($\approx 1.25 \times 10^3 \text{ J}$).

2. $t = E / P = 1246.5 / 50 = \mathbf{25 \text{ s}}$.

3. $Q = mc\Delta T = 0.018 \times 4180 \times 100 = 7524 \text{ J}$. Ratio = $7524 / 1246.5 = \mathbf{6.0}$. Water stores far more energy per kelvin because its molecules have many more ways to store energy.

Problem 19 Ice for a cold drink

Energy released by the water cooling to 0°C : $0.30 \times 4180 \times 25 = 31350 \text{ J}$.

Energy absorbed per kg of ice: warming $2100 \times 5 = 10\,500 \text{ J}$ plus melting $3.34 \times 10^5 \text{ J} = 344\,500 \text{ J}$.

$m = 31350 / 344\,500 = 0.0910 \text{ kg}$, so $m \approx \mathbf{0.091 \text{ kg (91 g)}}$.

Problem 20 Why hydrogen escapes

$T = M c^2 / (3R)$ with $c = 1.12 \times 10^4 \text{ m/s}$, so $c^2 = 1.254 \times 10^8 \text{ m}^2/\text{s}^2$.

1. $T = 0.0020 \times 1.254 \times 10^8 / (3 \times 8.31) = \mathbf{10063 \text{ K} \approx 1.0 \times 10^4 \text{ K}}$.

2. $T = 0.028 \times 1.254 \times 10^8 / (3 \times 8.31) = \mathbf{140887 \text{ K} \approx 1.4 \times 10^5 \text{ K}}$.

3. In a speed distribution a few hydrogen molecules in the fast tail exceed the escape speed even at atmospheric temperatures, so hydrogen leaks away over geological time. Nitrogen needs about fourteen times more energy, so almost none reaches escape speed.