

Emf, internal resistance and kirchhoff's laws

Physics grades 11-12 (A Level · IB · Class 11-12) · Lesson

Quick answer: The emf of a source is the energy it gives each coulomb of charge. A real source also has internal resistance r , so its terminal voltage is $V = \epsilon - Ir$. Kirchhoff's laws solve any network: current in equals current out at a junction, and the emfs equal the sum of the p.d.s around a loop.

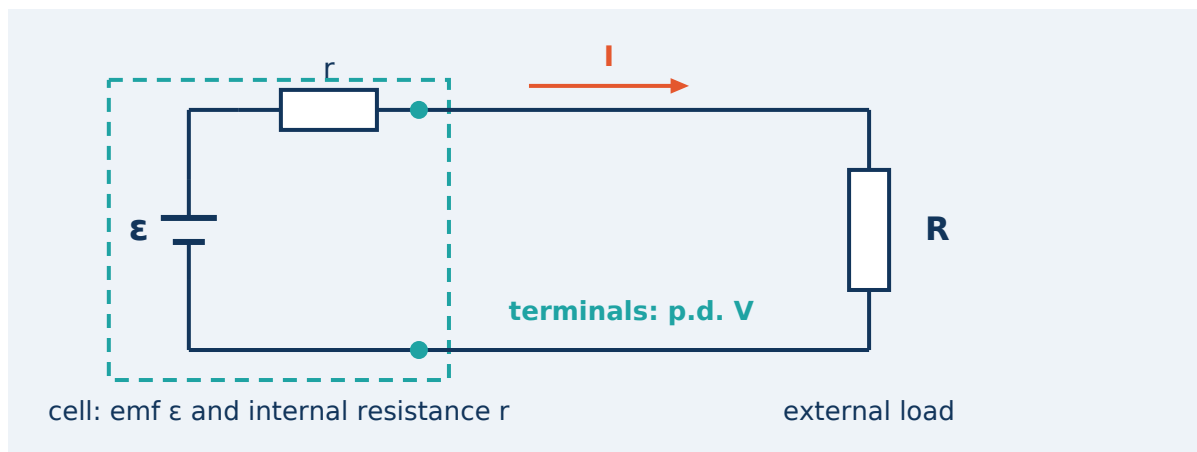
Level: Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB B.5

What are emf and internal resistance?

A battery does not deliver its full rated voltage when it powers a device. Some energy is wasted pushing charge through the cell itself, and the voltage at the terminals drops as the current grows.

Definition

Electromotive force (emf) ϵ is the energy transferred to each coulomb of charge by the source: $\epsilon = W / q$, measured in volts. The **internal resistance** r is the resistance of the source itself, in ohms.



A real cell behaves like an ideal emf ϵ in series with an internal resistance r . The terminal p.d. is $V = \epsilon - Ir$.

Formula

$$\epsilon = I(R + r) \text{ and } V = \epsilon - Ir$$

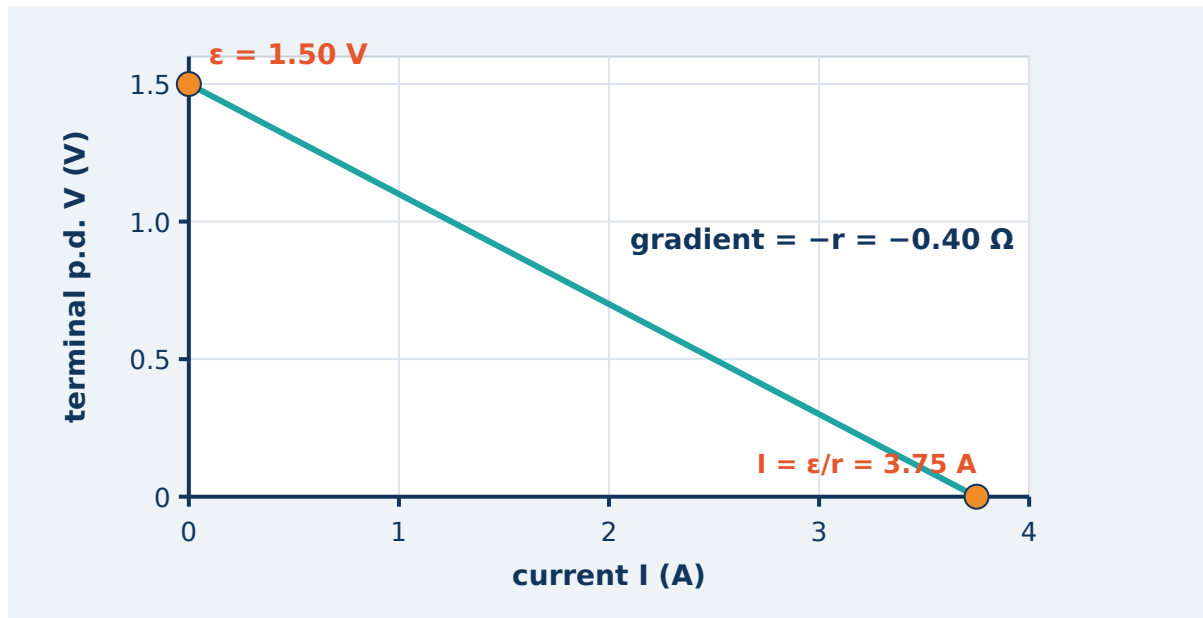
ϵ = emf (V), I = current (A), R = external resistance (Ω), r = internal resistance (Ω), V = terminal potential difference across the cell (V). The quantity Ir is the “lost volts”.

Worked example

A cell of emf 1.50 V and internal resistance 0.40 Ω is connected to a 2.6 Ω resistor. Find the current, the terminal p.d. and the lost volts.

$I = \epsilon / (R + r) = 1.50 / (2.6 + 0.40) = \mathbf{0.50 \text{ A}}$. Lost volts $= Ir = 0.50 \times 0.40 = \mathbf{0.20 \text{ V}}$.
Terminal p.d. $V = \epsilon - Ir = 1.50 - 0.20 = \mathbf{1.30 \text{ V}}$, which also equals $IR = 0.50 \times 2.6$.

Two limiting cases are useful. With no current (open circuit) there are no lost volts, so the terminal p.d. equals the emf. In a short circuit the external resistance is zero and the current is a maximum of ϵ / r .



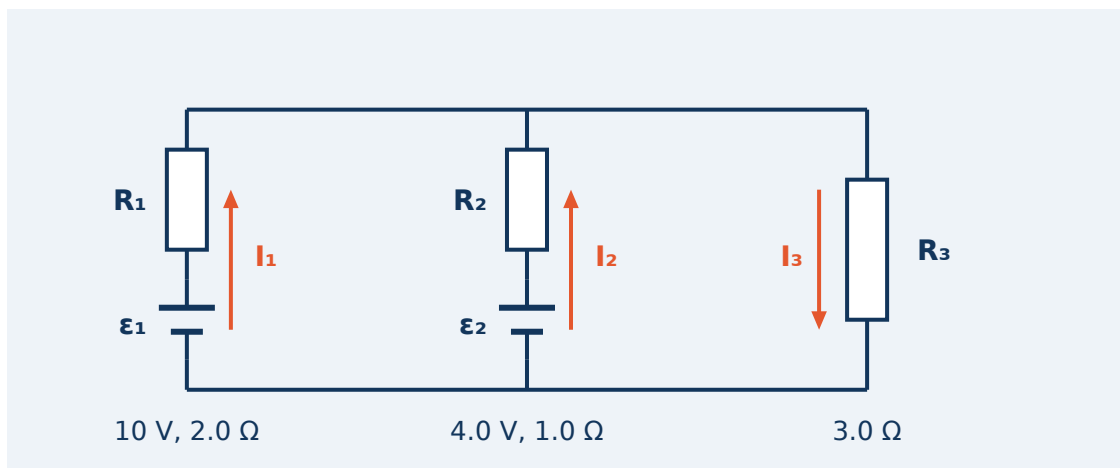
For a cell with $\epsilon = 1.50 \text{ V}$ and $r = 0.40 \Omega$, the line has intercept ϵ and gradient $-r$.

What do Kirchhoff's laws say?

Ohm's law is not enough for circuits with several branches or several sources. Two conservation laws solve any dc network.

Key law

First law (junction rule): the total current into a junction equals the total current out, which is conservation of charge. **Second law (loop rule):** around any closed loop the sum of the emfs equals the sum of the potential differences across the resistors, which is conservation of energy: $\Sigma \epsilon = \Sigma IR$.



Circuit for the worked example: I_1 and I_2 flow up through the cells and I_3 flows down through R_3 .

Worked example

In the circuit above, find the three currents.

Junction: $I_3 = I_1 + I_2$. Left loop: $10 = 2I_1 + 3I_3$ gives $10 = 5I_1 + 3I_2$. Middle loop: $4 = 1 \cdot I_2 + 3I_3$ gives $4 = 3I_1 + 4I_2$. Solving: $I_2 = -10/11 = -0.91 \text{ A}$, $I_1 = 28/11 = 2.55 \text{ A}$, $I_3 = 18/11 = 1.64 \text{ A}$. The negative sign shows that the 4.0 V cell is being charged, not discharged.

Common mistake

Do not assume that every cell pushes current out. Choose directions for the currents at the start, and if one answer comes out negative, the real current flows the other way.

How does a potential divider work?

Two resistors in series share the supply voltage in proportion to their resistances. The voltage across R_2 is $V_{\text{out}} = V_{\text{in}} \times R_2 / (R_1 + R_2)$. Replacing one resistor by a thermistor or a light-dependent resistor makes a sensor whose output changes with temperature or light.

Situation	Current	Terminal p.d.
Open circuit	0	$V = \varepsilon$
Normal load R	$I = \varepsilon / (R + r)$	$V = \varepsilon - Ir$
Short circuit	$I = \varepsilon / r$ (maximum)	$V = 0$
Load R equal to r	$I = \varepsilon / 2r$	$V = \varepsilon / 2$, maximum power transfer

Worked example

A 9.0 V supply is connected across 6.0 k Ω and 3.0 k Ω resistors in series. Find the voltage across the 3.0 k Ω resistor.

$V_{\text{out}} = 9.0 \times 3.0 / (6.0 + 3.0) = 3.0 \text{ V}$. The current is $9.0 / 9000 = 1.0 \text{ mA}$, which gives 3.0 V across 3.0 k Ω as a check.

Lina's tip

Before solving a network, redraw it with the current arrows and the $\pm$ ends of the cells marked. A clear sketch makes the loop equations almost write themselves.

Key points to remember

- Emf ε = energy per coulomb; terminal p.d. $V = \varepsilon - Ir$
- Current $I = \varepsilon / (R + r)$; short-circuit current = ε / r
- Kirchhoff 1: $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$; Kirchhoff 2: $\Sigma \varepsilon = \Sigma IR$
- Potential divider: $V_{\text{out}} = V_{\text{in}} \times R_2 / (R_1 + R_2)$
- Maximum power transfer happens when $R = r$

Emf, internal resistance and Kirchhoff's laws: frequently asked questions

What is the difference between emf and terminal potential difference?

The emf is the energy per coulomb the source supplies in total. The terminal p.d. is what is left across the external circuit after the energy lost in the internal resistance is subtracted, so $V = \mathcal{E} - Ir$.

Why does a battery voltage drop when you draw more current?

More current means a bigger voltage across the internal resistance, Ir . The terminal p.d. is the emf minus this lost voltage, so it falls as the current grows.

How can you measure internal resistance in a lab?

Measure the terminal p.d. for several currents and plot V against I . The intercept is the emf and the gradient is minus the internal resistance.

What does Kirchhoff's second law mean physically?

It is energy conservation. A charge that goes once around a closed loop returns to its starting potential, so the total energy gained from sources equals the total energy given up in resistors.

Why does a voltmeter change the reading in a potential divider?

A real voltmeter has a finite resistance, which sits in parallel with part of the divider. That lowers the resistance of that part and so lowers the voltage across it.