

# Emf, internal resistance and kirchhoff's laws

Physics grades 11-12 (A Level · IB · Class 11-12) · Worksheet answers

Each answer shows the formula, the substitution and the result with its unit. Check the sign of each emf and the direction of every current before you add up potential differences, and remember that small rounding differences are acceptable.

**Level:** Physics grades 11-12 (A Level · IB · Class 11-12) · **Curriculum:** IB B.5

## Problem 1 Lost volts

1. Lost volts =  $Ir = 0.60 \times 0.50 = \mathbf{0.30\ V}$ .
2.  $V = \epsilon - Ir = 1.5 - 0.30 = \mathbf{1.2\ V}$ .

## Problem 2 Current in a complete circuit

$$I = \epsilon / (R + r) = 12 / (5.8 + 0.20) = 12 / 6.0 = \mathbf{2.0\ A}.$$

## Problem 3 Junction rule

Current in =  $3.2 + 1.1 = 4.3\ \text{A}$ . Current out =  $2.4 + I$ . So  $I = 4.3 - 2.4 = \mathbf{1.9\ A}$ , by Kirchhoff's first law.

## Problem 4 Simple divider

1.  $I = V / R = 12 / 12\ 000 = \mathbf{1.0 \times 10^{-3}\ A\ (1.0\ mA)}$ .
2.  $V = IR$  gives  $\mathbf{4.0\ V}$  across the  $4.0\ \text{k}\Omega$  resistor and  $\mathbf{8.0\ V}$  across the  $8.0\ \text{k}\Omega$  resistor, which add to  $12\ \text{V}$ .

## Problem 5 Ideas and definitions

1. Emf is the **energy transferred to each coulomb of charge by the source** ( $\epsilon = W / q$ ).
2. Charge must pass through the internal resistance, so some energy per coulomb,  $Ir$ , is transferred inside the battery. The terminal p.d. is  $\mathbf{V = \epsilon - Ir}$ .
3. **Energy** is conserved: around a closed loop the emfs equal the sum of the p.d.s.

## Problem 6 Jump-start accident

1.  $I = \epsilon / r = 12.6 / 0.020 = \mathbf{630\ A}$ .
2. A current of hundreds of amperes dissipates  $I^2r$  in the battery and heats the spanner and leads very quickly. The result can be burns, fire or an exploding battery.

### Problem 7 Open circuit and load

1. With no current the terminal p.d. equals the emf, so  $\mathcal{E} = 1.58 \text{ V}$ .
2.  $I = V / R = 1.40 / 8.0 = 0.175 \text{ A}$ .
3. Lost volts =  $1.58 - 1.40 = 0.18 \text{ V}$ .  $r = 0.18 / 0.175 = 1.03 \Omega \approx 1.0 \Omega$ .

### Problem 8 Power bank

1.  $V = \mathcal{E} - Ir = 5.0 - 2.0 \times 0.15 = 4.7 \text{ V}$ .
2.  $P = VI = 4.7 \times 2.0 = 9.4 \text{ W}$ .
3.  $P_{\text{lost}} = I^2 r = 2.0^2 \times 0.15 = 0.60 \text{ W}$ . Efficiency =  $9.4 / (5.0 \times 2.0) = 94 \%$ .

### Problem 9 Cells in series or in parallel

1. Series:  $\mathcal{E} = 3.0 \text{ V}$ ,  $r = 1.0 \Omega$ , so  $I = 3.0 / (2.0 + 1.0) = 1.0 \text{ A}$ .
2. Parallel:  $\mathcal{E} = 1.5 \text{ V}$ ,  $r = 0.50 / 2 = 0.25 \Omega$ , so  $I = 1.5 / (2.0 + 0.25) = 0.67 \text{ A}$ .
3. The series arrangement gives the larger current because it doubles the emf, while the load resistance is much larger than the internal resistance.

### Problem 10 Meter loading

1.  $V = 9.0 \times 10 / 20 = 4.5 \text{ V}$ .
2.  $1/R = 1/10 + 1/10 = 0.2 \text{ k}\Omega^{-1}$ , so  $R = 5.0 \text{ k}\Omega$ .
3. The total is  $10 + 5.0 = 15 \text{ k}\Omega$ , so the reading is  $9.0 \times 5.0 / 15 = 3.0 \text{ V}$ , much lower than the true  $4.5 \text{ V}$ .

### Problem 11 Light sensor divider

1.  $V = 6.0 \times 1.0 / (4.7 + 1.0) = 1.05 \text{ V}$ .
2.  $V = 6.0 \times 100 / (4.7 + 100) = 5.73 \text{ V}$ .
3. The LDR resistance rises sharply in the dark, so it takes a larger share of the supply voltage and the output rises from about  $1 \text{ V}$  to nearly  $6 \text{ V}$ .

### Problem 12 Battery feeding two lamps

1.  $1/R = 1/6.0 + 1/3.0 = 0.5$ , so  $R = 2.0 \Omega$ .
2.  $I = \mathcal{E} / (R + r) = 9.0 / (2.0 + 1.0) = 3.0 \text{ A}$ .
3.  $V = \mathcal{E} - Ir = 9.0 - 3.0 \times 1.0 = 6.0 \text{ V}$ .  $I = V / R$  gives  $1.0 \text{ A}$  in the  $6.0 \Omega$  lamp and  $2.0 \text{ A}$  in the  $3.0 \Omega$  lamp.

### Problem 13 Reading a cell graph

1. The line meets the  $V$  axis at  $I = 0$ , so  $\mathcal{E} = 1.55 \text{ V}$ .
2. Gradient =  $\Delta V / \Delta I = (1.15 - 1.55) / (1.0 - 0) = -0.40 \Omega$ .
3. The gradient equals  $-r$ , so  $r = 0.40 \Omega$ . At  $0.50 \text{ A}$ ,  $V = 1.55 - 0.50 \times 0.40 = 1.35 \text{ V}$ .

### Problem 14 Opposing cells

1. The cells oppose, so the net emf =  $9.0 - 3.0 = \mathbf{6.0\ V}$ .
2.  $I = 6.0 / (2.0 + 4.0) = \mathbf{1.0\ A}$ , flowing in the direction set by the 9.0 V cell.
3.  $V = IR$  gives  $\mathbf{2.0\ V}$  and  $\mathbf{4.0\ V}$ . Check:  $9.0 - 3.0 = 2.0 + 4.0$ , so both sides equal 6.0 V.

### Problem 15 Spot the error

1. The internal resistance was left out of the current calculation, and the terminal p.d. was set equal to the emf even though a current flows.
2.  $I = \varepsilon / (R + r) = 6.0 / (5.0 + 1.0) = \mathbf{1.0\ A}$ .
3.  $V = \varepsilon - Ir = 6.0 - 1.0 \times 1.0 = \mathbf{5.0\ V}$  (which equals  $IR = 1.0 \times 5.0$ ).

### Problem 16 Two sources, one load

1.  $I_3 = I_1 + I_2$ . Left loop:  $12 = 3I_1 + 6I_3 = 9I_1 + 6I_2$ . Right loop:  $9.0 = 2I_2 + 6I_3 = 6I_1 + 8I_2$ .
2. Multiply the first loop by 4 and the second by 3:  $48 = 36I_1 + 24I_2$  and  $27 = 18I_1 + 24I_2$ . Subtracting:  $21 = 18I_1$ , so  $I_1 = \mathbf{1.17\ A}$ . Then  $12 = 10.5 + 6I_2$  gives  $I_2 = \mathbf{0.25\ A}$ , and  $I_3 = \mathbf{1.42\ A}$ .
3.  $V = I_3R = 1.417 \times 6.0 = \mathbf{8.5\ V}$ .  $P = I_3^2R = 2.007 \times 6.0 = \mathbf{12.0\ W}$ .

### Problem 17 Maximum power transfer

1.  $I = \varepsilon / (R + r)$  and  $P = I^2R = \varepsilon^2R / (R + r)^2$ .
2.  $P = 144 \times 0.50 / (2.5)^2 = \mathbf{11.52\ W}$ ;  $P = 144 \times 2.0 / (4.0)^2 = \mathbf{18.0\ W}$ ;  $P = 144 \times 8.0 / (10)^2 = \mathbf{11.52\ W}$ .
3. Maximum power occurs when  $\mathbf{R = r = 2.0\ \Omega}$ , with  $P_{\max} = \varepsilon^2 / 4r = 144 / 8.0 = \mathbf{18\ W}$ . At that point the load and the internal resistance share the power equally, so the efficiency is  $\mathbf{50\ \%}$ .

### Problem 18 Charging a battery

1. The emfs oppose, so  $I = (18.0 - 12.0) / (6.0 + 1.0) = \mathbf{0.857\ A \approx 0.86\ A}$ .
2. Panel:  $V = 18.0 - 0.857 \times 6.0 = 12.86\ V$ . Battery:  $V = 12.0 + 0.857 \times 1.0 = 12.86\ V$ . The two are **equal (12.86 V)**, as they must be across the same terminals.
3. Chemical power =  $\varepsilon I = 12.0 \times 0.857 = \mathbf{10.3\ W}$ . Heat =  $I^2(6.0 + 1.0) = 0.735 \times 7.0 = \mathbf{5.1\ W}$ . Check:  $18.0 \times 0.857 = 15.4\ W = 10.3 + 5.1$ .

### Problem 19 Balancing a bridge

1.  $V_x = 10 \times 6.0 / (4.0 + 6.0) = \mathbf{6.0\ V}$ .
2. The reading is zero when  $V_y = 6.0\ V$ :  $10R / (3.0 + R) = 6.0$ , so  $10R = 18 + 6R$ ,  $4R = 18$  and  $\mathbf{R = 4.5\ k\Omega}$ .
3.  $V_y = 10 \times 5.0 / (3.0 + 5.0) = 6.25\ V$ . The reading is  $6.25 - 6.0 = \mathbf{0.25\ V}$ , with Y at the higher potential.

**Problem 20 Linearising the cell equation**

1.  $V = IR$  and  $I = \varepsilon / (R + r)$  give  $V = \varepsilon R / (R + r)$ . Inverting:  $1/V = (R + r) / (\varepsilon R) = 1/\varepsilon + (r/\varepsilon)(1/R)$ .

2.  $1/R = 1.00, 0.50, 0.25, 0.10 \Omega^{-1}$ ;  $1/V = 1.000, 0.833, 0.750, 0.700 \text{ V}^{-1}$ .

3. Gradient =  $(1.000 - 0.700) / (1.00 - 0.10) = 0.333 \Omega/\text{V} = r/\varepsilon$ . Intercept =  $0.667 \text{ V}^{-1} = 1/\varepsilon$ .

So  $\varepsilon = 1.5 \text{ V}$  and  $r = 0.333 \times 1.5 = 0.50 \Omega$ .