

Resistance, Kirchhoff's rules and steady-state RC circuits

AP Physics 2 (algebra-based) · Lesson

Quick answer: In a DC circuit, a wire's resistance is $R = \rho L/A$, series resistors add, and parallel resistors share the same voltage. Kirchhoff's junction rule conserves charge and his loop rule conserves energy. In steady state a capacitor carries no current, so the circuit is solved with resistors only.

Level: AP Physics 2 (algebra-based) · **Curriculum:** AP P2 Unit 11

How does resistivity set the resistance of a wire?

Charge carriers in a wire keep colliding with the lattice of atoms. A longer wire means more collisions, a wider wire offers more parallel paths, and the material decides how strong each collision effect is. These three ideas combine into one relationship.

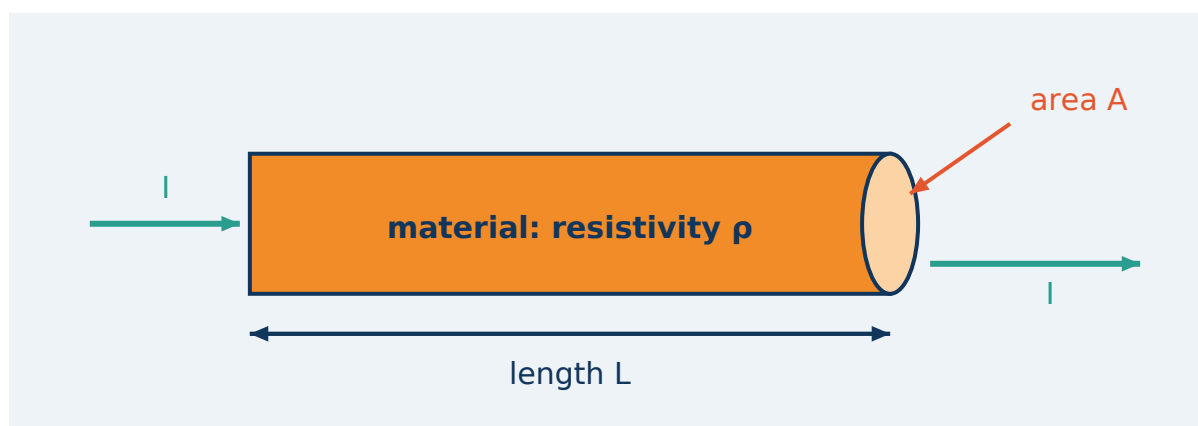
Definition

Resistivity (ρ) is a property of a material, measured in ohm metres ($\Omega \cdot \text{m}$). It does not depend on the size of the sample. **Resistance** (R) belongs to one particular piece of wire and depends on its shape too.

Formula

$$R = \rho L / A, \text{ and } V = IR$$

R = resistance (Ω), ρ = resistivity ($\Omega \cdot \text{m}$), L = length (m), A = cross-sectional area (m^2), V = potential difference (V), I = current (A).



A longer wire resists more, a thicker wire resists less, and the material sets the scale.

Worked example

Aarav in Pune uses a copper lead 50 m long with diameter 1.0 mm. Copper has $\rho = 1.7 \times$

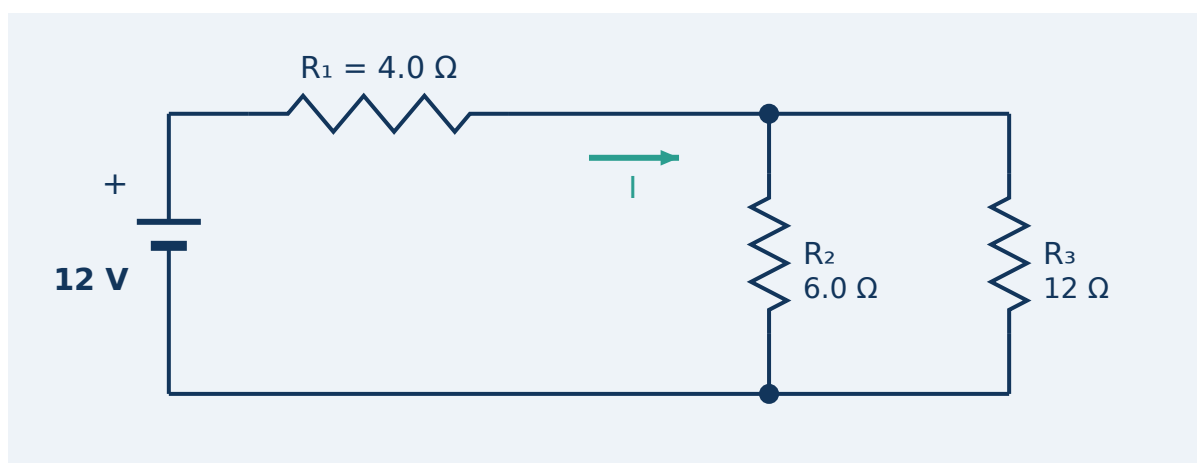
$10^{-8} \Omega \cdot \text{m}$. Find its resistance.

$A = \pi(d/2)^2 = \pi(0.50 \times 10^{-3} \text{ m})^2 = 7.85 \times 10^{-7} \text{ m}^2$. Then $R = \rho L/A = (1.7 \times 10^{-8})(50) / (7.85 \times 10^{-7}) = \mathbf{1.1 \Omega}$.

How do resistors combine in series and in parallel?

Resistors in series share one path, so the same current passes through each. Resistors in parallel share two junctions, so the same potential difference lies across each. The table gives the rules, and the circuit below shows both arrangements at once.

Feature	Series	Parallel
Current	same in every resistor	splits between the branches
Potential difference	splits between the resistors	same across every branch
Equivalent resistance	$R_{\text{eq}} = R_1 + R_2 + \dots$	$1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \dots$
Adding one more resistor	R_{eq} increases	R_{eq} decreases



One resistor in series with a parallel pair: the pair has less resistance than either branch alone.

Worked example

In the circuit, $\varepsilon = 12 \text{ V}$, $R_1 = 4.0 \Omega$, $R_2 = 6.0 \Omega$ and $R_3 = 12 \Omega$. Find the current in each resistor.

Parallel pair: $1/R_p = 1/6.0 + 1/12 = 3/12$, so $R_p = 4.0 \Omega$. Total $R_{\text{eq}} = 4.0 + 4.0 = 8.0 \Omega$. The battery current is $I = 12 / 8.0 = 1.5 \text{ A}$, which is the current in R_1 . The pair has $V = IR_p = 1.5 \times 4.0 = 6.0 \text{ V}$, so $I_2 = 6.0 / 6.0 = \mathbf{1.0 \text{ A}}$ and $I_3 = 6.0 / 12 = \mathbf{0.50 \text{ A}}$. They add up to 1.5 A.

What are Kirchhoff's two rules?

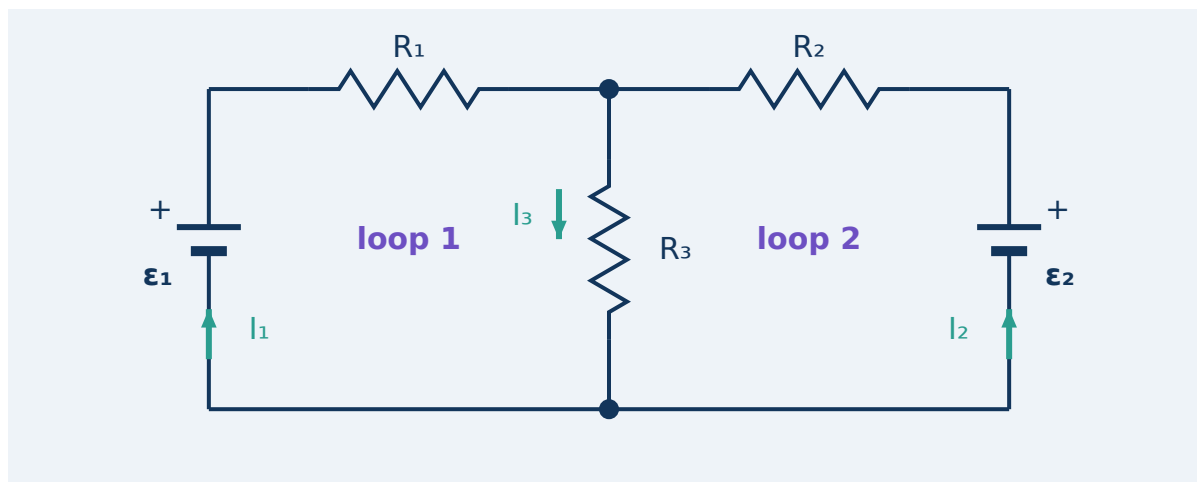
Key law

Junction rule: the total current entering a junction equals the total current leaving it (conservation of charge).

Loop rule: the potential changes around any closed loop add up to zero (conservation of

energy).

Choose a direction for each current. Going round a loop, a resistor crossed along the current is a drop of IR , and crossed against it is a rise of IR . A battery crossed from its negative to its positive side is a rise of $\mathcal{E}$. If you finish with a negative current, the real direction is opposite to the one you chose.



Choose current directions, write one junction equation, then one loop equation for each independent loop.

Worked example

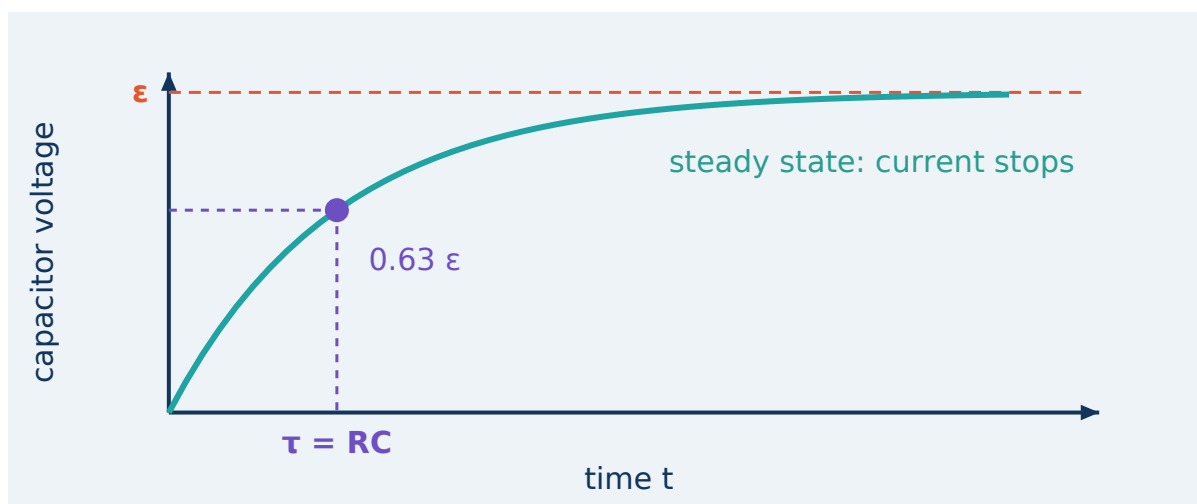
Take $\mathcal{E}_1 = 12 \text{ V}$, $\mathcal{E}_2 = 6.0 \text{ V}$, $R_1 = 2.0 \Omega$, $R_2 = 1.0 \Omega$ and $R_3 = 4.0 \Omega$ in the two-loop circuit.

Junction: $I_3 = I_1 + I_2$. Loop 1: $12 - 2.0I_1 - 4.0I_3 = 0$. Loop 2: $6.0 - 1.0I_2 - 4.0I_3 = 0$.

Substituting I_3 gives $6I_1 + 4I_2 = 12$ and $4I_1 + 5I_2 = 6$. Solving: $I_1 = \mathbf{2.6 \text{ A}}$, $I_2 = \mathbf{-0.86 \text{ A}}$, $I_3 = \mathbf{1.7 \text{ A}}$. The negative sign means the 6.0 V battery is being charged: current flows down through it.

What happens in a steady-state RC circuit?

A capacitor stores charge $Q = CV$. At the instant a circuit is switched on, an uncharged capacitor behaves like a plain wire, so the current is large. As charge builds up, its voltage opposes the battery and the current falls. The time scale is the time constant $\tau = RC$, and after one τ the capacitor voltage has reached about 63 % of its final value.



The capacitor voltage rises quickly at first, then levels off at the emf.

Common mistake

Do not say "no current means no voltage" for a charged capacitor. In steady state its branch carries zero current, yet its voltage equals the voltage across whatever it is connected in parallel with.

Worked example

A 12 V battery drives $R_1 = 3.0 \text{ k}\Omega$ in series with $R_2 = 1.0 \text{ k}\Omega$. A capacitor $C = 47 \text{ }\mu\text{F}$ is connected across R_2 . After a long time, find its charge.

The capacitor branch carries no current, so $I = 12 / 4000 = 3.0 \text{ mA}$. The voltage across R_2 is $3.0 \text{ mA} \times 1.0 \text{ k}\Omega = 3.0 \text{ V}$, which is also V_C . Then $Q = CV = (47 \times 10^{-6})(3.0) = \mathbf{1.4 \times 10^{-4} \text{ C}}$.

Lina's tip

Redraw the circuit twice. At $t = 0$ replace the uncharged capacitor with a wire. In steady state replace it with a gap. Both redrawn circuits only need the resistor rules.

Key points to remember

- Resistance of a wire: $R = \rho L/A$, with ρ in $\Omega \cdot \text{m}$
- Series: $R_{\text{eq}} = R_1 + R_2 + \dots$; parallel: $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \dots$
- Junction rule: $\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$; loop rule: $\Sigma \Delta V = 0$ around a closed loop
- Capacitor: $Q = CV$; in steady state its branch current is zero
- Time constant $\tau = RC$; about 63 % charged after one τ

Resistance, Kirchhoff's rules and steady-state RC circuits: frequently asked questions

What is the difference between resistance and resistivity?

Resistivity is a property of a material and has units of $\Omega \cdot \text{m}$. Resistance depends on resistivity

and also on the length and area of the particular wire, and is measured in ohms.

Why does adding a resistor in parallel lower the total resistance?

Each new branch gives the current another path. More paths mean more total current for the same voltage, so the equivalent resistance falls below that of the smallest branch.

What does a negative current mean in Kirchhoff's rules?

It only means the real current flows opposite to the direction you chose at the start. The equations are still correct, and you can quote the magnitude with the reversed direction.

How does a capacitor behave in steady state?

After a long time it is fully charged and no current flows into its branch, so it acts like an open gap. Its voltage equals the voltage across the parallel part of the circuit it belongs to.

What is the time constant of an RC circuit?

It is $\tau = RC$, measured in seconds. After one time constant a charging capacitor has reached about 63 % of its final voltage, and after five it is almost fully charged.