

Moment of inertia by integration and torque

AP Physics C (Mechanics and E&M, calculus-based) · Test answers (mark scheme)

Model answers with a marking guide. Give method marks for a correct set-up of the integral or the equations of motion even when the arithmetic fails, and apply error carried forward when later parts use earlier wrong values. Units are required for every final answer.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 5

Question 1 Moment of inertia (2 marks)

a. $I = \int r^2 dm$, the sum of each mass element times the square of its perpendicular distance from the axis. b. $\Sigma \tau = I\alpha$.

Marking: 1 mark each.

Question 2 Rod by integration (3 marks)

a. $dm = (M/L) dx$. b. $I = (M/L) \int_0^L x^2 dx = ML^2/3$. c. $I = 0.50 \times 0.36/3 = \mathbf{0.060 \text{ kg}\cdot\text{m}^2}$.

Marking: 1 mark for each part.

Question 3 Parallel axis (3 marks)

a. $I_{cm} = 0.50 \times 0.36/12 = 0.015 \text{ kg}\cdot\text{m}^2$. b. $I = I_{cm} + Md^2$. c. $I = 0.015 + 0.50 \times 0.040 = \mathbf{0.035 \text{ kg}\cdot\text{m}^2}$.

Marking: 1 mark each.

Question 4 Pulley and mass (4 marks)

a. $mg - T = ma$; $TR = I\alpha$ with $I = \frac{1}{2}MR^2$ and $\alpha = a/R$, so $T = \frac{1}{2}Ma$. b. $a = mg/(m + M/2) = 14.7/2.5 = \mathbf{5.9 \text{ m/s}^2}$. c. $T = \frac{1}{2}Ma = 1.0 \times 5.89 = \mathbf{5.9 \text{ N}}$. d. $\alpha = a/R = 5.89/0.10 = \mathbf{59 \text{ rad/s}^2}$.

Marking: 1 mark per part.

Question 5 Braking a flywheel (3 marks)

a. $\alpha = \tau/I = 20/0.80 = \mathbf{25 \text{ rad/s}^2}$. b. $t = \omega/\alpha = 50/25 = \mathbf{2.0 \text{ s}}$. c. $K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.80 \times 2500 = \mathbf{1.0 \times 10^3 \text{ J}}$.

Marking: 1 mark per part.

Question 6 Hoop and disc (2 marks)

The **disc** arrives first. The hoop has the larger moment of inertia (MR^2 against $\frac{1}{2}MR^2$), so a

larger share of the energy is rotational, leaving less for translation and giving a smaller acceleration.

Marking: 1 mark for the disc; 1 mark for the I explanation.

Question 7 Spinning skater (2 marks)

No external torque acts, so $L = I\omega$ is constant. Reducing I forces ω up. If I halves, ω **doubles**.

Marking: 1 mark for conservation of L ; 1 mark for the factor 2.

Question 8 Disc formula (1 mark)

$I = \frac{1}{2}MR^2$.

Marking: 1 mark.