

Moment of inertia by integration and torque

AP Physics C (Mechanics and E&M, calculus-based) · Worksheet

Name: _____ Date: _____ Class: _____

These problems practise the three ideas of this unit: finding moment of inertia by summing or integrating, shifting the axis with the parallel axis theorem, and applying $\Sigma \tau = I \alpha$ with angular momentum. They begin with direct substitution and end with derivations and linked pulley and rolling problems for AP Physics C: Mechanics unit 5.

Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 5

Problem 1 Dumbbell inertia (easy)

Two 0.50 kg masses are fixed at the ends of a light rod, each 0.40 m from the axis through the rod's centre.

1. Calculate the moment of inertia.

Problem 2 Units (easy)

1. Give the SI unit of moment of inertia in base units.
2. Give the SI unit of torque and explain why it is not called joule.

Problem 3 Torque from a spanner (easy)

Chiamaka in Lagos pulls a spanner of length 0.25 m with a 30 N force.

1. Calculate the torque when the force is perpendicular to the spanner.
2. Calculate the torque when the force makes 30° with the spanner.

Problem 4 Angular acceleration (easy)

A torque of 6.0 N·m acts on a wheel with $I = 0.40 \text{ kg}\cdot\text{m}^2$.

1. Calculate the angular acceleration.

Problem 5 Using the table (easy)

A solid sphere of mass 5.0 kg and radius 0.10 m, and a thin hoop of the same mass and radius.

1. Calculate I for the sphere about a diameter.
2. Calculate I for the hoop about its central axis.

Problem 6 Ring against disc (easy)

A solid disc and a ring have the same mass and radius, and the same torque acts on each.

1. State which has the larger moment of inertia.
2. State which spins up faster and explain.

Problem 7 Rod about two axes (medium)

A uniform rod has $M = 0.60 \text{ kg}$ and $L = 1.0 \text{ m}$.

1. Calculate I about its centre.
2. Calculate I about one end using the parallel axis theorem.

Problem 8 Disc about the rim (medium)

A uniform disc has $M = 2.0 \text{ kg}$ and $R = 0.20 \text{ m}$.

1. Calculate I about its central axis.
2. Calculate I about a parallel axis through the rim.

Problem 9 Rod with varying density (medium)

A rod of length 1.5 m has linear density $\lambda = 2.0x^2 \text{ kg/m}$, where x is measured from one end.

1. Calculate the mass of the rod.
2. Calculate I about the end where $x = 0$.

Problem 10 Flywheel on a rope (medium)

Mei in Singapore pulls a tangential rope on a solid disc of mass 6.0 kg and radius 0.30 m with a steady force of 12 N .

1. Calculate the torque.
2. Calculate I .
3. Calculate α and the angular speed after 2.5 s from rest.

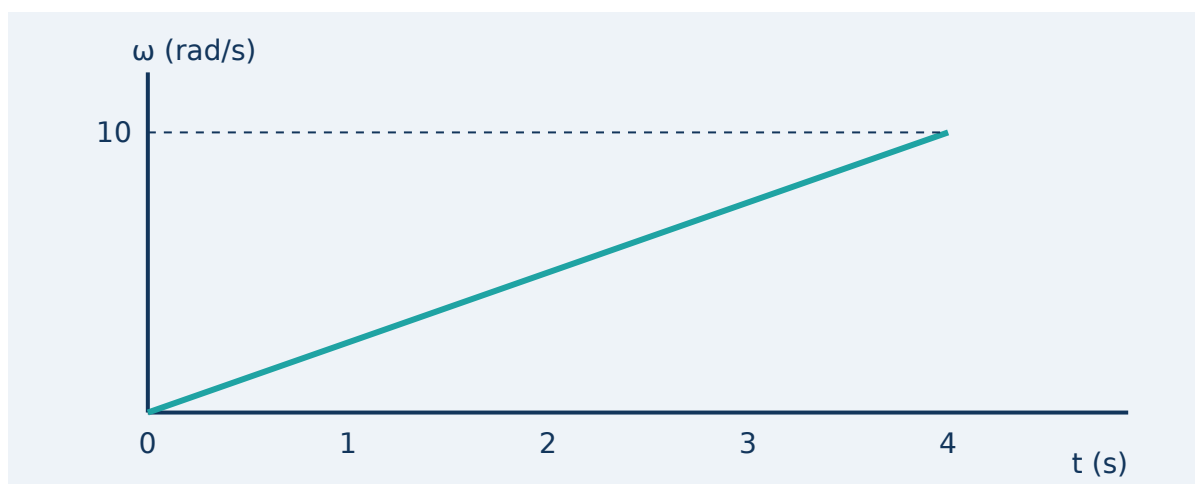
Problem 11 Skater spinning (medium)

A skater with arms out has $I = 3.0 \text{ kg}\cdot\text{m}^2$ and $\omega = 4.0 \text{ rad/s}$. She pulls her arms in so $I = 1.2 \text{ kg}\cdot\text{m}^2$.

1. Calculate her new angular speed.
2. Compare the kinetic energies before and after.

Problem 12 Reading ω against t (medium)

The graph shows the angular speed of a flywheel with $I = 0.50 \text{ kg}\cdot\text{m}^2$ as a motor accelerates it from rest.



Angular speed of the flywheel.

1. Calculate α from the gradient.
2. Calculate the torque.
3. Calculate the angle turned in 4.0 s.

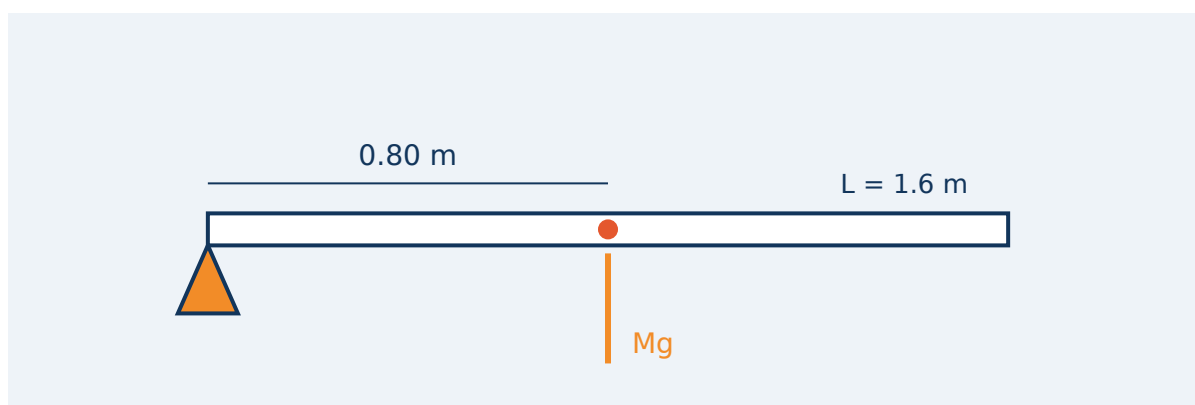
Problem 13 Plank pivot point (medium)

A uniform plank has mass 8.0 kg and length 2.4 m. Diego in Texas drills a pivot hole 0.40 m from one end.

1. Calculate the distance d from the hole to the centre of mass.
2. Calculate I about an axis through the hole, perpendicular to the plank.
3. A classmate adds $M(0.40)^2$ to $ML^2/3$. Explain why that gives a wrong answer.

Problem 14 Swinging signboard bar (medium)

A uniform bar of mass 2.5 kg and length 1.6 m is hinged at one end, held level and then let go.



Bar hinged at one end.

1. Calculate its moment of inertia about the hinge.
2. Calculate the angular acceleration at the instant of release.
3. Calculate the upward force of the hinge at that instant.

Problem 15 Disc by integration (hard)

Derive the moment of inertia of a uniform disc of mass M and radius R about its central axis.

1. Write dm for a ring of radius r .
2. Integrate.
3. Calculate I for $M = 0.80$ kg and $R = 0.15$ m.

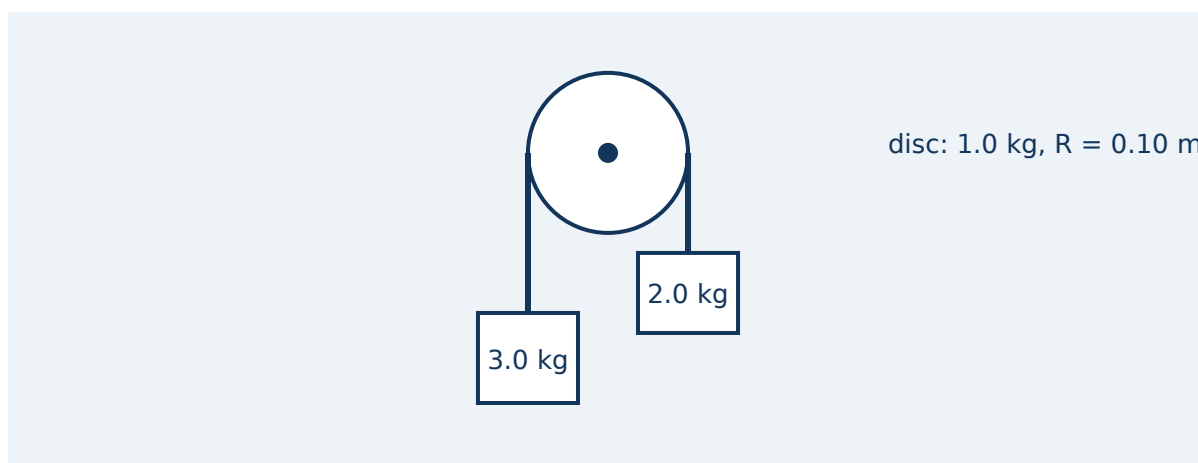
Problem 16 Annular disc (hard)

A uniform flat annulus has inner radius $R_1 = 0.10$ m, outer radius $R_2 = 0.20$ m and mass 3.0 kg.

1. Show that $I = \frac{1}{2}M(R_1^2 + R_2^2)$.
2. Calculate I .

Problem 17 Massive pulley (hard)

Aarav sets up a pulley (solid disc, $M = 1.0$ kg, $R = 0.10$ m) with masses of 3.0 kg and 2.0 kg on a light rope that does not slip.

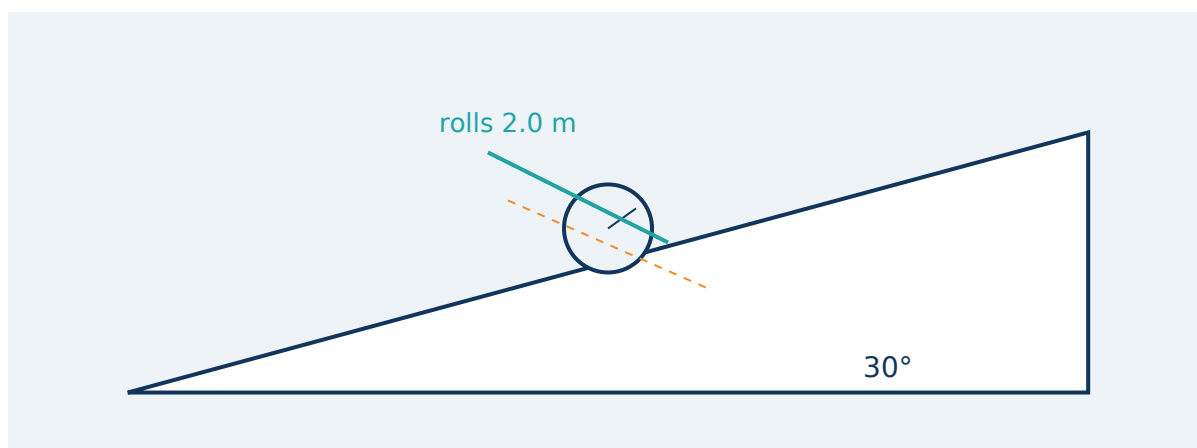


Massive pulley with two masses.

1. Find the acceleration of the masses.
2. Find the two tensions.

Problem 18 Rolling cylinder (hard)

A solid cylinder rolls without slipping down a 30° incline.



Cylinder rolling down a slope.

1. Show $a = g \sin \theta / (1 + I/MR^2)$ and calculate a .
2. Calculate the speed after rolling 2.0 m from rest.

Problem 19 Compound pendulum arm (hard)

A uniform rod ($M = 0.60$ kg, $L = 1.0$ m) has a small ball of mass 0.30 kg, treated as a point mass, attached at one end. It is pivoted at the other end.

1. Calculate I about the pivot.
2. Calculate the initial angular acceleration when released from horizontal.

Problem 20 Clay hits a disc (hard)

A free disc ($M = 1.0$ kg, $R = 0.30$ m) can rotate about its central axis. A 0.050 kg clay ball moving at 6.0 m/s tangentially hits the rim and sticks.

1. Calculate the angular momentum of the clay about the axis.
2. Calculate the final angular speed.
3. Calculate the kinetic energy lost.