

Energy levels, radioactive decay and binding energy

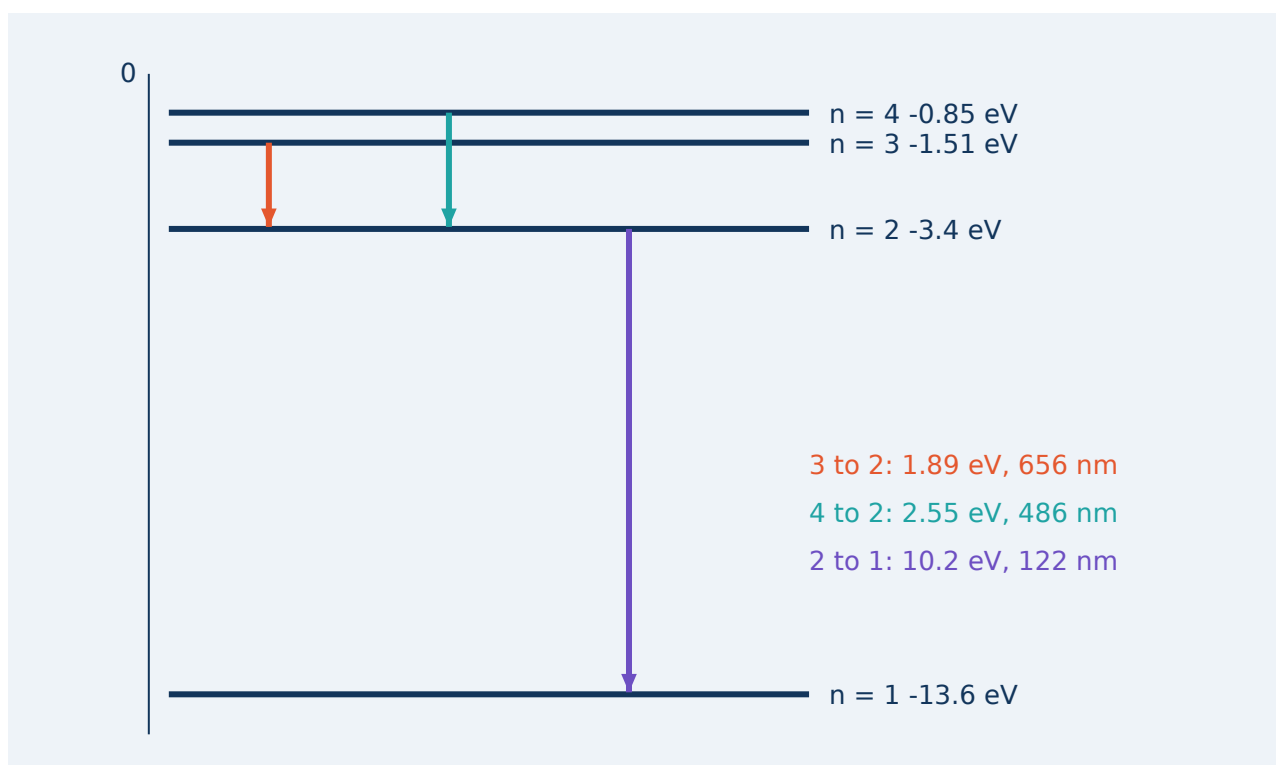
AP Physics 2 (algebra-based) · Lesson

Quick answer: Atoms have discrete energy levels, so they emit and absorb photons of fixed energy equal to the gap between levels. Unstable nuclei decay randomly by alpha, beta or gamma emission, and half the nuclei decay in each half-life. A nucleus has less mass than its separate nucleons, and the mass defect gives the binding energy through $E = mc^2$.

Level: AP Physics 2 (algebra-based) · **Curriculum:** AP P2 Unit 15

How do energy levels explain atomic spectra?

Electrons in an atom can only have certain energies. In hydrogen these energies are $E = -13.6 \text{ eV} / n^2$, where $n = 1, 2, 3, \dots$ and the energy is negative because the electron is bound. An atom emits a photon when the electron drops to a lower level, and the photon energy equals the gap between the two levels. Because only certain gaps exist, the light from a gas contains only certain wavelengths: a line spectrum.



Hydrogen energy levels with three emission transitions.

Definition

Emission line spectrum: the set of discrete wavelengths emitted by an excited gas, each line matching one transition between energy levels. An absorption spectrum has dark lines at the same wavelengths.

Formula

$$E_{\text{photon}} = E_{\text{initial}} - E_{\text{final}} = hf = hc / \lambda$$

Energies are in eV, h is the Planck constant, f the frequency (Hz) and λ the wavelength (nm when $hc = 1240 \text{ eV}\cdot\text{nm}$). An atom absorbs a photon only when its energy equals the gap between two levels.

Worked example

A hydrogen electron falls from $n = 3$ to $n = 2$. Data: $E_3 = -1.51 \text{ eV}$, $E_2 = -3.40 \text{ eV}$.

Method: $\Delta E = -1.51 - (-3.40) = 1.89 \text{ eV}$, then $\lambda = 1240 / 1.89$. Answer: **656 nm**, which is visible red light.

What is inside a nucleus and how does it decay?

A nucleus contains protons and neutrons. Its symbol ${}^A_Z X$ has mass number A (protons plus neutrons) and atomic number Z (protons). Unstable nuclei decay at random, and the decay of any single nucleus cannot be predicted. For a large sample, the fraction that decays in a given time is fixed.

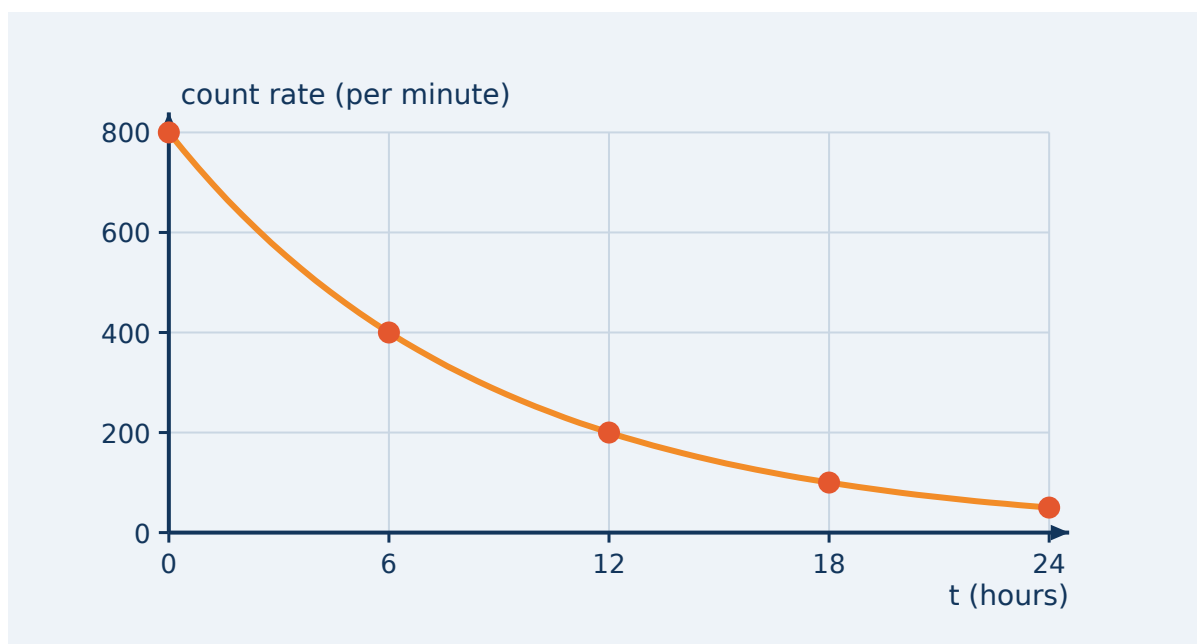
Radiation	What it is	Change to nucleus	Stopped by
Alpha (α)	helium nucleus ${}^4_2\text{He}$	A falls by 4, Z falls by 2	paper or a few cm of air
Beta minus (β^-)	electron (and an antineutrino)	A same, Z rises by 1	a few mm of aluminium
Gamma (γ)	high-energy photon	none, energy lost	thick lead or concrete

Key law

In every nuclear equation the total mass number A and the total charge Z on the left equal those on the right. Example: ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He}$.

What is a half-life?

The half-life T is the time for half the nuclei in a sample to decay. After each half-life, the number of nuclei left, and the activity, are halved again. It does not depend on how many nuclei are present or on temperature.



Exponential decay: each 6 hours the rate halves.

Worked example

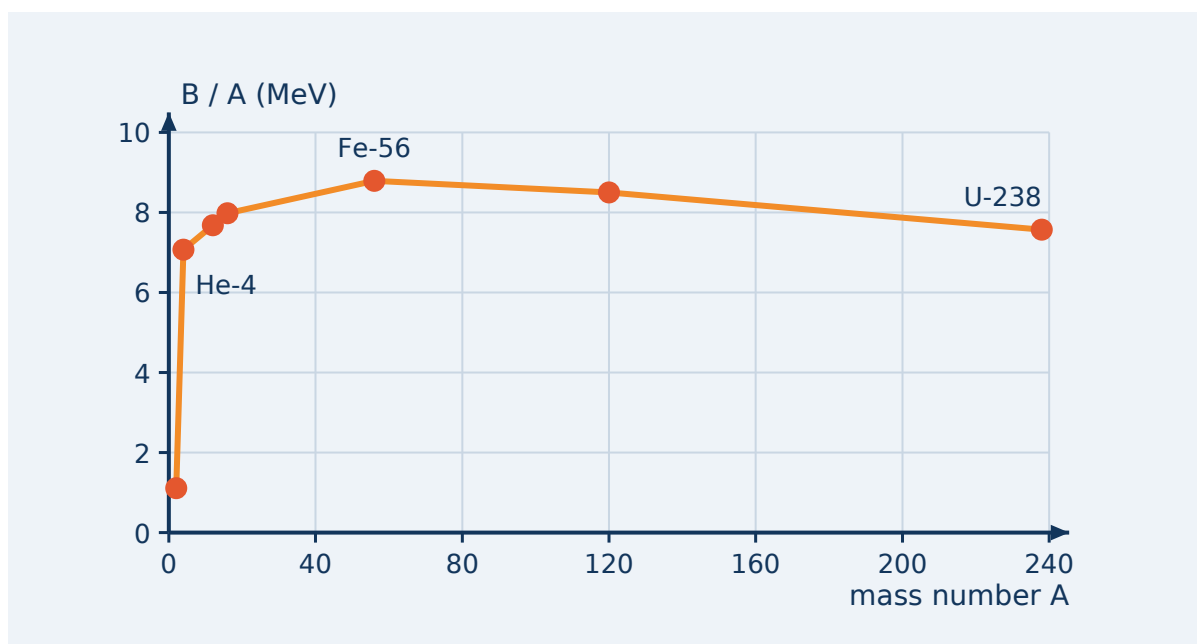
A hospital has 80 mg of iodine-131, which has a half-life of 8.0 days. Data: 24 days is 3 half-lives. Method: $80 \rightarrow 40 \rightarrow 20 \rightarrow 10$. Answer: **10 mg** remains, which is $(\frac{1}{2})^3 = \frac{1}{8}$ of the original.

Common mistake

After two half-lives, *one quarter* remains, not zero. The sample never reaches zero by halving; it just becomes smaller and smaller. Also, do not say that 2 half-lives equals $2 \times$ half of the sample.

How are mass and energy related in nuclear reactions?

The mass of a nucleus is slightly less than the combined mass of its separate protons and neutrons. This difference is the mass defect Δm . The energy that would be needed to take the nucleus apart is the binding energy, $E = \Delta m c^2$. With the atomic mass unit, 1 u corresponds to 931.5 MeV.



Binding energy per nucleon peaks near iron-56.

The curve rises steeply and peaks near iron-56. Light nuclei release energy when they fuse, and very heavy nuclei release energy when they split, because both processes move toward the more tightly bound middle of the curve.

Worked example

Helium-4 nucleus: $\Delta m = 2(1.007276) + 2(1.008665) - 4.001506 = 0.030376 \text{ u}$. Method: $E = 0.030376 \times 931.5 \text{ MeV}$. Answer: **28.3 MeV** in total, which is 7.07 MeV per nucleon.

Lina's tip

Before you do a nuclear equation, count the top numbers and the bottom numbers on each side separately. Two quick additions catch nearly every mistake.

Key points to remember

- Hydrogen levels: $E = -13.6 \text{ eV} / n^2$. Photon energy equals the gap between levels.
- Nuclear equations conserve mass number A and atomic number Z.
- Half-life T: the amount left after time t is $N = N_0(1/2)^{(t / T)}$.
- Binding energy $E = \Delta m c^2$, and 1 u corresponds to 931.5 MeV.
- Binding energy per nucleon peaks near iron-56, so fusion and fission both release energy.

Energy levels, radioactive decay and binding energy: frequently asked questions

Why do atoms emit only certain wavelengths?

Electrons can only occupy certain energy levels, so a transition releases a photon with exactly the energy gap between two levels. Each gap gives one wavelength, so the spectrum is a set

of lines rather than a continuous band.

What is the difference between alpha, beta and gamma radiation?

Alpha is a helium nucleus and is stopped by paper. Beta is a fast electron and is stopped by a few millimetres of aluminium. Gamma is a high-energy photon and needs thick lead or concrete to reduce it.

Does the half-life depend on the size of the sample?

No. Each nucleus has the same chance of decaying in a given time, so half of any sample decays in one half-life, whether it contains a few nuclei or billions.

What is mass defect?

Mass defect is the difference between the total mass of the separate nucleons and the mass of the nucleus they form. It is converted into the binding energy that holds the nucleus together, $E = \Delta m c^2$.

Why does fusion release energy for light nuclei but not for heavy ones?

Binding energy per nucleon rises with mass number up to iron-56. Fusing light nuclei moves them up the curve, which releases energy. Beyond iron, fusion would need energy, so splitting heavy nuclei releases it instead.