

Energy levels, radioactive decay and binding energy

AP Physics 2 (algebra-based) · Worksheet answers

Each answer lists the relationship, the substitution and the result with its unit. For nuclear equations, check that the top numbers and the bottom numbers each add up on both sides before you accept an answer.

Level: AP Physics 2 (algebra-based) · **Curriculum:** AP P2 Unit 15

Problem 1 Counting protons and neutrons

1. Protons = $Z = 6$, neutrons = $14 - 6 = 8$.
2. Protons = 92, neutrons = $235 - 92 = 143$.

Problem 2 Hydrogen energy levels

1. **-13.6 eV, -3.40 eV, -1.51 eV, -0.85 eV.**
2. The electron must reach 0 eV, so the energy needed is **3.40 eV**.

Problem 3 A visible hydrogen line

1. $\Delta E = -0.85 - (-3.40) = 2.55 \text{ eV}$.
2. $\lambda = 1240 / 2.55 = 486 \text{ nm}$, a blue-green line.

Problem 4 Absorbing a photon

1. $\Delta E = -3.40 - (-13.6) = 10.2 \text{ eV}$.
2. **No.** An absorbed photon must match a gap exactly, and 11.0 eV does not match 10.2 eV ($n = 2$) or 12.1 eV ($n = 3$).

Problem 5 Alpha decay equation

1. ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^4_2\text{He}$.
2. **A = 222, Z = 86.**

Problem 6 Beta decay equation

1. ${}^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni} + {}^0_{-1}\text{e} + \bar{\nu}$ (antineutrino).
2. **Z = 28:** a neutron changes into a proton.

Problem 7 Counting half-lives

- 15 days is 3 half-lives: $64 \rightarrow 32 \rightarrow 16 \rightarrow \mathbf{8.0\ g}$.
- 20 days is 4 half-lives: **4.0 g**.
- Fraction = $(\frac{1}{2})^3 = \mathbf{1/8}$.

Problem 8 Reading a decay graph

- The rate halves from 800 to 400 in 6 hours, so **T = 6.0 h**.
- 30 h is 5 half-lives: $800 / 32 = \mathbf{25\ per\ minute}$.
- Each half-life removes half of what is left, and decay is random, so a few nuclei always remain.

Problem 9 Mass to energy

- $E = mc^2 = 1.0 \times 10^{-3} \times (3.00 \times 10^8)^2 = \mathbf{9.0 \times 10^{13}\ J}$.
- $9.0 \times 10^{13} / 4.2 \times 10^9 = \mathbf{2.1 \times 10^4}$, about 21 000 tonnes of TNT.

Problem 10 Unit conversion for mass defect

- $E = 0.0050 \times 931.5 = \mathbf{4.7\ MeV}$.
- $4.66\ MeV \times 1.60 \times 10^{-13} = \mathbf{7.5 \times 10^{-13}\ J}$.

Problem 11 Identify the radiation

- Paper stops almost nothing, so there is **no significant alpha**. Aluminium cuts the rate a lot, so **beta** is present. Lead removes the rest, so **gamma** is present too.
- Beta $\approx (1190 - 400) / 1200 \approx \mathbf{66\ \%}$; gamma $\approx 400 / 1200 \approx \mathbf{33\ \%}$.

Problem 12 Carbon dating

- 12.5 % is $(\frac{1}{2})^3$, so the age is $3 \times 5730 = \mathbf{17\ 190\ years}$.
- 30 % lies between 50 % (one half-life) and 25 % (two half-lives), so the age is between **5730 and 11 460 years**.
- $t = T \ln(0.30) / \ln(0.5) = 5730 \times 1.737 = \mathbf{9.95 \times 10^3\ years}$.

Problem 13 Why spectra are lines

- Electrons in the gas atoms can only be at fixed energy levels, so transitions release photons with only certain energies, giving certain wavelengths.
- The cool gas absorbs photons whose energies match its level gaps, removing those wavelengths from the continuous light and leaving **dark lines** at the same wavelengths as the emission lines.

Problem 14 Lyman and Balmer lines

- 10.2 eV and **1.89 eV**.
- $1240 / 10.2 = \mathbf{122 \text{ nm}}$ and $1240 / 1.89 = \mathbf{656 \text{ nm}}$.
- The 122 nm line is **ultraviolet**; the 656 nm line is **visible (red)**.

Problem 15 Binding energy of the deuteron

- $\Delta m = 1.007276 + 1.008665 - 2.013553 = \mathbf{0.002388 \text{ u}}$.
- $E = 0.002388 \times 931.5 = \mathbf{2.22 \text{ MeV}}$.
- $2.22 / 2 = \mathbf{1.11 \text{ MeV per nucleon}}$.

Problem 16 Energy from radium decay

- $\Delta m = 226.025410 - (222.017578 + 4.002603) = \mathbf{0.005229 \text{ u}}$.
- $E = 0.005229 \times 931.5 = \mathbf{4.87 \text{ MeV}}$.
- The alpha receives $222 / 226$ of the energy: $4.87 \times 222 / 226 = \mathbf{4.78 \text{ MeV}}$.

Problem 17 Activity of a medical tracer

- $T = 6.0 \times 3600 = \mathbf{2.16 \times 10^4 \text{ s}}$.
- $A = 0.693 / 2.16 \times 10^4 \times 2.0 \times 10^{10} = \mathbf{6.4 \times 10^5 \text{ Bq}}$.
- 18 h is 3 half-lives, so $A = 6.4 \times 10^5 / 8 = \mathbf{8.0 \times 10^4 \text{ Bq}}$.

Problem 18 Power of the Sun

- $\Delta m = 4 \times 1.007825 - 4.002603 = 4.031300 - 4.002603 = \mathbf{0.028697 \text{ u}}$.
- $E = 0.028697 \times 931.5 = \mathbf{26.7 \text{ MeV}}$.
- $m = P / c^2 = 3.8 \times 10^{26} / 9.0 \times 10^{16} = \mathbf{4.2 \times 10^9 \text{ kg per second}}$.

Problem 19 Lines from a hypothetical atom

- Transitions: $4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$, $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$, giving **6 lines**.
- The smallest gap is $4 \rightarrow 3$, 1.0 eV, so $\lambda = 1240 / 1.0 = \mathbf{1240 \text{ nm (infrared)}}$.
- The largest gap is $4 \rightarrow 1$, 7.5 eV, so $\lambda = 1240 / 7.5 = \mathbf{165 \text{ nm}}$.

Problem 20 Half-life from lab data

- Half of 480 is 240, which occurs between 2 min (276) and 3 min (209): interpolating gives about **2.5 min**.
- 120 is a quarter of 480, which means two half-lives in 5 min, so **T = 2.5 min**.
- Background counts do not come from the source and do not decay, so they would distort the ratio of counts and give a wrong half-life.