

Orbits, escape speed and gravitational energy

AP Physics C (Mechanics and E&M, calculus-based) · Lesson

Quick answer: Gravitational potential energy is $U = -GMm/r$, with zero at infinity. A circular orbit has $v = \sqrt{GM/r}$ and total energy $E = -GMm/2r$. A projectile escapes if its total energy is zero or more, giving an escape speed of $\sqrt{2GM/R}$.

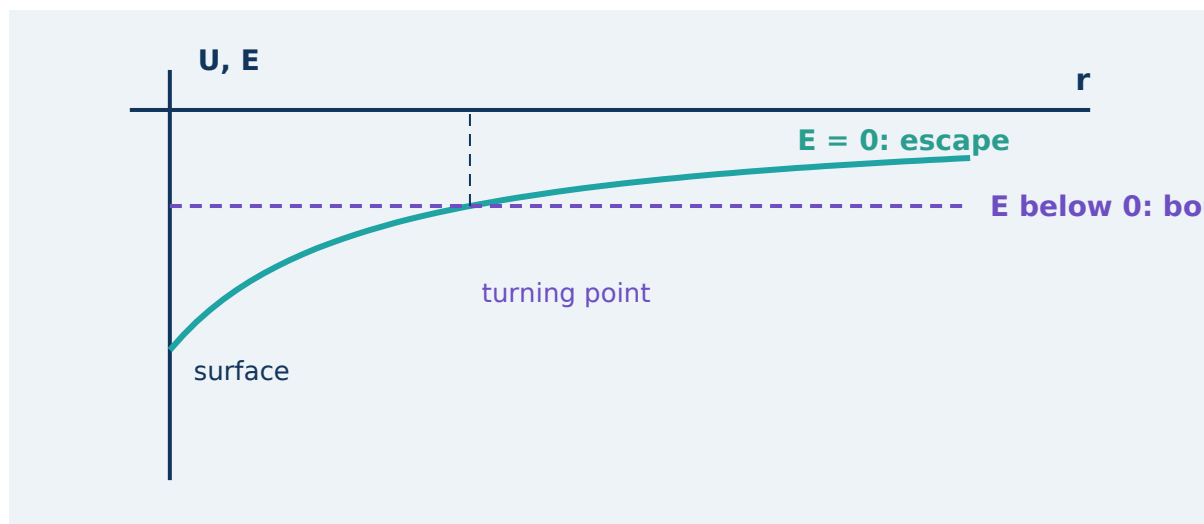
Level: AP Physics C (Mechanics and E&M, calculus-based) · **Curriculum:** AP PC:M Unit 7

What is gravitational potential energy?

Near the ground we use $U = mgh$, but that assumes a constant g . Across large distances Newton's law of gravitation, $F = GMm/r^2$, weakens with distance, so the potential energy must be found by integration. Taking $U = 0$ at infinite separation, the work needed to bring a mass in from infinity is negative, so U is always negative for a bound system.

Definition

The gravitational potential energy of two masses M and m , with their centres a distance r apart, is $U(r) = -\int$ from ∞ to r of $F \, dr = -GMm/r$. The force is $F = -dU/dr$, and $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

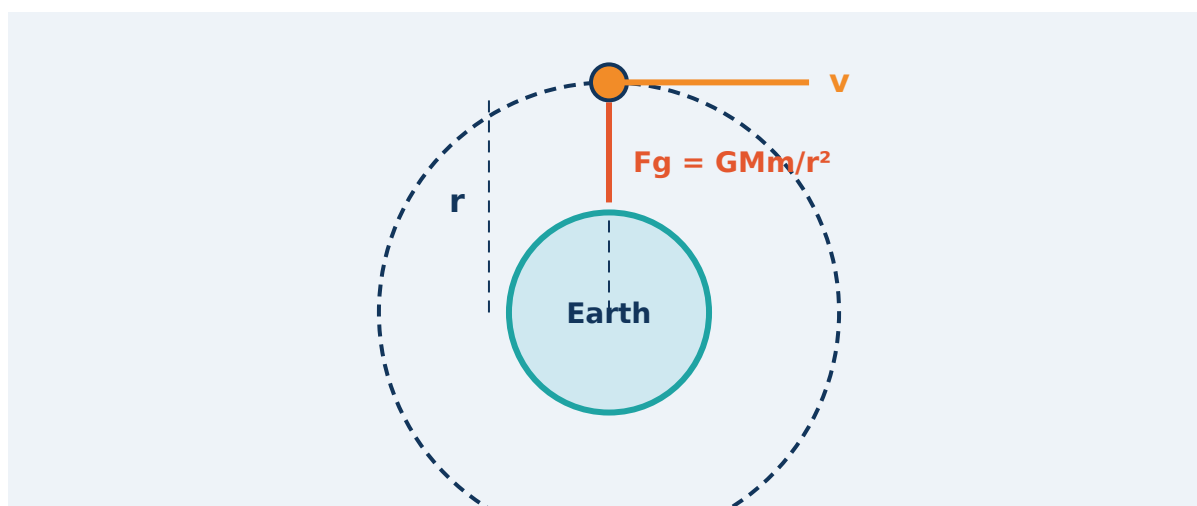


Potential energy well: $U = -GMm/r$. A total energy of zero or more means escape.

The curve is a well. A system with total energy $E = K + U$ below zero is bound and turns back at a maximum distance. If $E \geq 0$ it never turns back.

How fast does a satellite orbit?

For a circular orbit gravity is the only force, and it provides the centripetal acceleration: $GMm/r^2 = m v^2/r$. Solving gives the speed, and the period follows from $T = 2\pi r/v$.



Circular orbit: gravity supplies the centripetal force, perpendicular to the velocity.

Formula

$v = \sqrt{GM/r}$ and $T = 2\pi\sqrt{r^3/GM}$. Here $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$, M is the mass of the central body in kg, r is the orbit radius from the centre in m, v is in m/s and T in s. Squaring the period gives Kepler's third law: $T^2 = (4\pi^2/GM) r^3$.

Quantity (circular orbit of radius r)	Expression	Depends on satellite mass?
Orbital speed	$v = \sqrt{GM/r}$	No
Period	$T = 2\pi\sqrt{r^3/GM}$	No
Kinetic energy	$K = GMm/2r$	Yes
Potential energy	$U = -GMm/r = -2K$	Yes
Total energy	$E = -GMm/2r = -K$	Yes

The energy results are worth remembering: $K = -E$ and $U = 2E$. A satellite that loses energy to air drag moves to a lower orbit, where it travels faster, because $K = -E$ increases as E becomes more negative.

Worked example 1

A satellite orbits 400 km above Earth, so $r = 6.37 \times 10^6 + 4.0 \times 10^5 = 6.77 \times 10^6 \text{ m}$. Using $GM = 3.98 \times 10^{14} \text{ N m}^2/\text{kg}^2$ for Earth, $v = \sqrt{(3.98 \times 10^{14} / 6.77 \times 10^6)} = \mathbf{7.67 \text{ km/s}}$. The period is $T = 2\pi(6.77 \times 10^6) / 7670 = 5.55 \times 10^3 \text{ s}$, which is **92.5 minutes**.

What is escape speed?

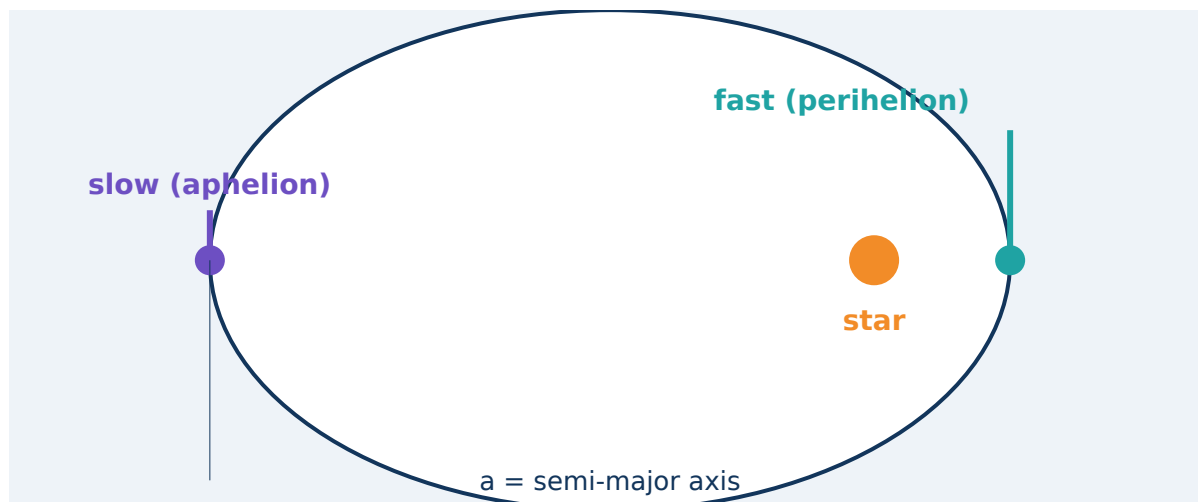
To escape, a projectile needs total energy zero: $\frac{1}{2}mv^2 - GMm/R = 0$. The mass cancels, so the escape speed from a body of radius R is $v_{\text{esc}} = \sqrt{2GM/R}$. It is $\sqrt{2}$ times the circular orbital speed at the same radius.

Worked example 2

From Earth's surface, $v_{\text{esc}} = \sqrt{(2 \times 3.98 \times 10^{14} / 6.37 \times 10^6)} = \sqrt{(1.25 \times 10^8)} = \mathbf{11.2}$

km/s. From the Moon ($M = 7.35 \times 10^{22}$ kg, $R = 1.74 \times 10^6$ m) the same method gives **2.37 km/s**, which is why the Moon holds no thick atmosphere.

How do elliptical orbits and Kepler's laws fit in?



Kepler's laws: the planet sweeps equal areas in equal times, so it is fastest at perihelion.

Gravity is a central force, so the torque about the star is zero and angular momentum $L = m v r_{\perp}$ is conserved. This gives Kepler's second law (equal areas in equal times, $dA/dt = L/2m$) and says that v is greatest where r is smallest. For an ellipse with semi-major axis a , the total energy is $E = -GMm/2a$ and the period is $T = 2\pi\sqrt{a^3/GM}$.

Common mistake

Do not use $U = mgh$ for a satellite, and do not put the orbital height into the formula in place of r . Always measure r from the centre of the planet, so $r = R + \text{altitude}$.

Lina's tip

To check a gravity answer quickly, remember that E for a circular orbit is minus the kinetic energy. If your total energy comes out positive for a satellite that stays in orbit, you have made a sign error.

Key points to remember

- Gravitational force $F = GMm/r^2$; potential energy $U = -GMm/r$ (zero at infinity).
- Circular orbit: $v = \sqrt{GM/r}$, $T = 2\pi\sqrt{r^3/GM}$, $K = -E = GMm/2r$.
- Escape speed: $v_{\text{esc}} = \sqrt{2GM/R}$, independent of the projectile's mass.
- Elliptical orbit: $E = -GMm/2a$; angular momentum is conserved, so v is greatest at the closest point.
- Kepler's third law: $T^2 = (4\pi^2/GM)a^3$.

Orbits, escape speed and gravitational energy: frequently asked questions

Why is gravitational potential energy negative?

Because we choose $U = 0$ at infinite separation. Gravity is attractive, so bringing the masses closer releases energy, and the system energy falls below zero. A negative total energy means the system is bound.

Why does a satellite speed up when it loses energy?

Air drag lowers the total energy $E = -GMm/2r$, so r decreases. A smaller r means a larger orbital speed $v = \sqrt{GM/r}$, and kinetic energy $K = -E$ rises. The satellite spirals inward while gaining speed.

Does escape speed depend on the mass of the rocket?

No. The kinetic energy needed grows with the rocket's mass, but so does its gravitational potential energy, so the mass cancels in $\frac{1}{2}mv^2 = GMm/R$. The escape speed depends only on the planet's mass and radius.

Why do astronauts float if gravity still acts on them?

Both the station and the astronaut are in free fall around Earth with the same acceleration. At 400 km, g is about 8.7 m/s^2 , roughly 89 % of its surface value, but no contact force from the floor is needed, so they feel weightless.

What is the link between Kepler's second law and angular momentum?

The gravitational force always points to the star, so it exerts no torque about it. Angular momentum $m v r_{\perp}$ stays constant, which means the area swept per second, $dA/dt = \frac{1}{2} r v_{\perp}$, is also constant.